The *Smashed Filter* for Compressive Classification and Target Recognition

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Joint work with

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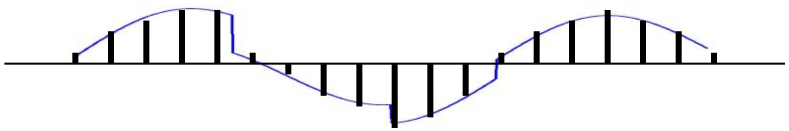
dsp.rice.edu/cs

Data Explosion

- DSP revolution:
  sample first and ask questions later

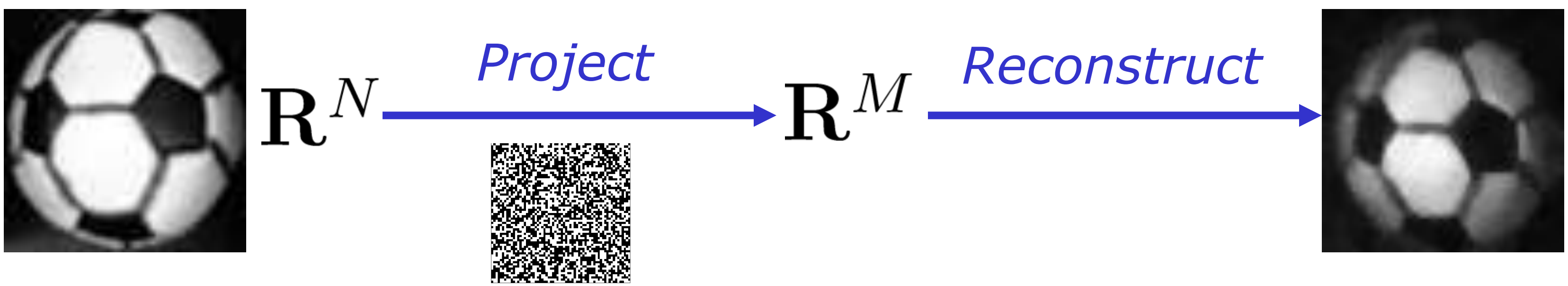


- Increasing ***pressure*** on classification algorithms
  - ever faster training and classification rates
  - ever larger, higher-dimensional data
  - ever lower energy consumption
  - radically new sensing modalities

- How can we acquire and process high-dimensional data quickly and efficiently?

Compressive Classification

- ***Random projections** preserve information*
  - Johnson-Lindenstrauss Lemma (point clouds – 1984)
  - Compressed Sensing (sparse signals – CRT, Donoho – 2004)



- If we can reconstruct a signal from compressive measurements, we should be able to perform
  - detection
  - classification
  - estimation
  - …

Multiclass Likelihood Ratio Test

- Observe one of *P* known signals in noise

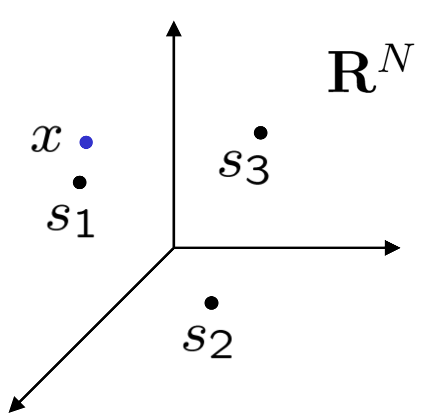
$$
\begin{aligned}
H_1 &: x = s_1 + n \\
H_2 &: x = s_2 + n \\
&\vdots \\
H_P &: x = s_P + n
\end{aligned}
$$

- Classify according to:

$$\arg\max_{j=1,\ldots,P} \; p(x|H_j)$$

- AWGN: *nearest-neighbor* classification

$$\arg\min_{j=1,\ldots,P} \; \|x - s_j\|_2$$



Johnson-Lindenstrauss Lemma

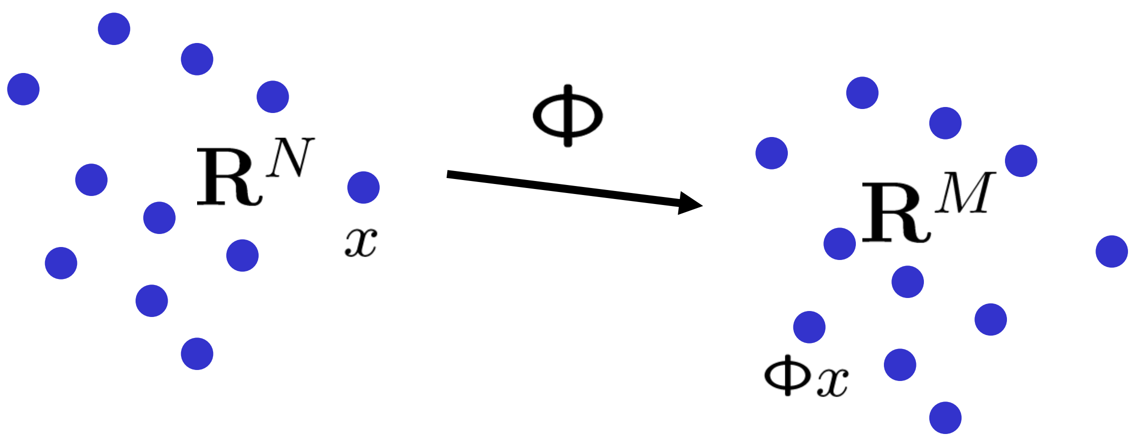
Let $\epsilon \in (0,1)$ be given. For every set Q of $|Q|$ points in $\mathbb{R}^N$, if

$$M = O\left(\frac{\log(|Q|/\delta)}{\epsilon^2}\right),$$

a randomly drawn $M \times N$ matrix Φ will satsify

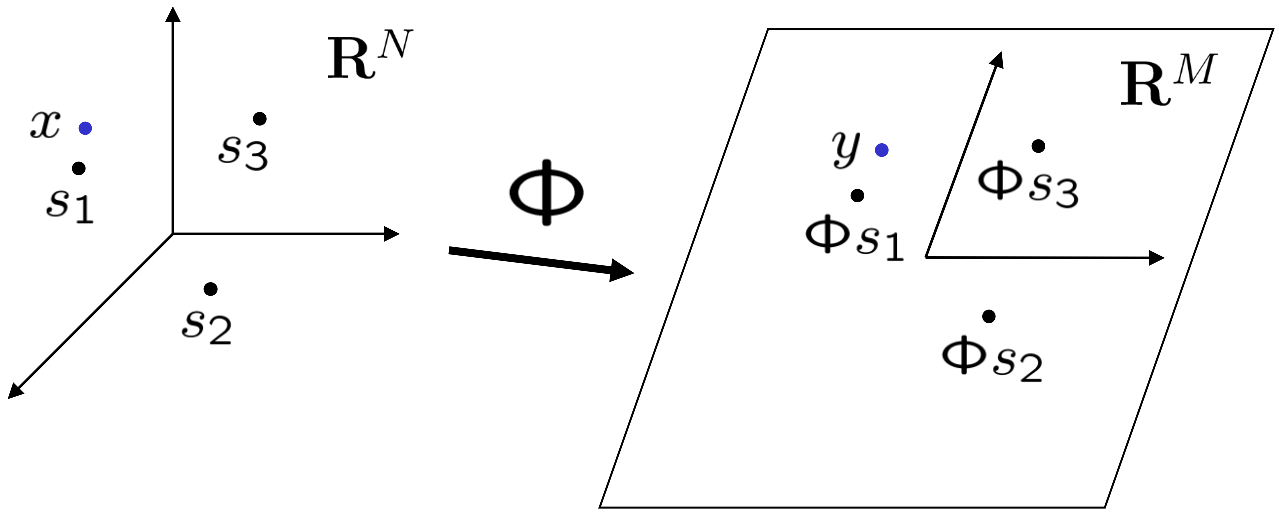
$$(1-\epsilon)\|u-v\|_2 \le \|\Phi u - \Phi v\|_2 \le (1+\epsilon)\|u-v\|_2$$

for all $u, v \in Q$ with probability at least $1-\delta$.



Compressive LRT

- Now suppose we observe $H_j \; : \; y = \Phi(s_j + n)$



$$t_1 = \|y - \Phi s_1\|_2$$
$$t_2 = \|y - \Phi s_2\|_2$$
$$t_3 = \|y - \Phi s_3\|_2$$

by the JL Lemma these distances are preserved

[Waagen 05, Davenport 06, Haupt 06]

Matched Filters

- We may know what signals we are looking for, but we may not know ***where*** to look

$$H_j \ : \ x = s_j(t - \theta_j) + n$$

- Elegant solution: matched filter

Compute

$$\langle x, s_j(t - \theta_j) \rangle \text{ for all } \theta_j$$

$$\Updownarrow$$

$$x * s_j(-t)$$

Challenge: Modify the compressive LRT to accommodate ***unknown parameters***

Generalized Likelihood Ratio Test

- GLRT:

$$\underset{j=1,\ldots,P}{\arg\max} \; p(x|\widehat{\theta}_j, H_j)$$

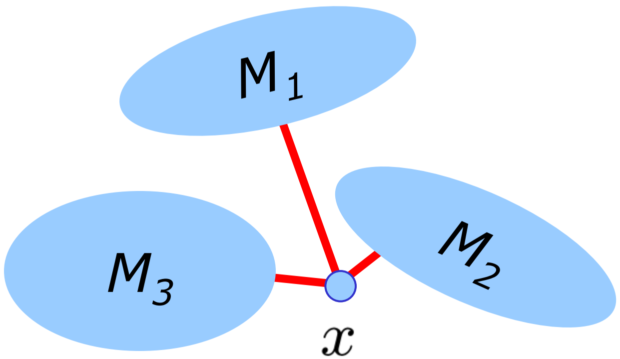
where

$$\widehat{\theta}_j = \underset{\theta \in \Theta_j}{\arg\max} \; p(x|\theta, H_j)$$

- Matched filter is a special case of the GLRT
- GLRT approach can be extended to any case where each class can be ***parameterized***
- If mapping from parameters to signal is well-behaved, each class forms a ***manifold***

Manifold Classification

- Now suppose our data is drawn from one of *P* possible manifolds:



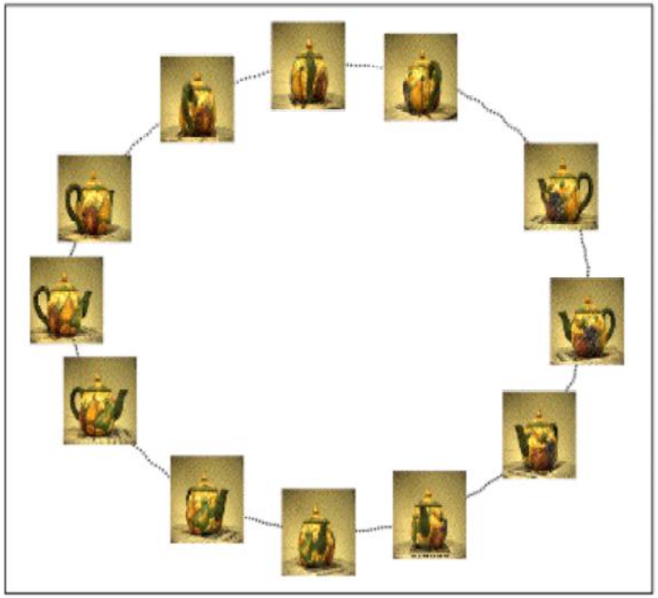
$$H_j \; : \; x = m_j + n, \qquad m_j \in M_j$$

$$m_j = f_j(\theta_j)$$

$$\underset{j=1,\ldots,P}{\arg\max} \; p(x|\widehat{\theta}_j, H_j)$$

$$\widehat{\theta}_j = \underset{\theta \in \Theta_j}{\arg\min} \; \|x - f_j(\theta_j)\|_2$$

$$\underset{j=1,\ldots,P}{\arg\min} \; \|x - f_j(\widehat{\theta}_j)\|_2$$

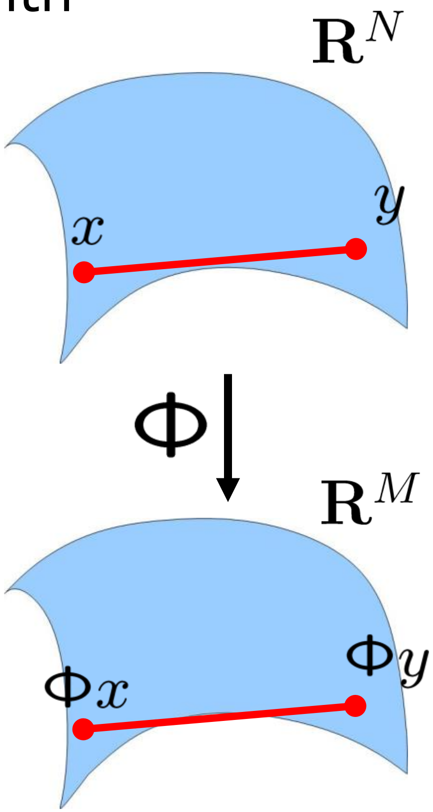


Stable Manifold Embedding

Theorem:

Let $F \subset R^N$ be a compact *K-dimensional manifold* with

- condition number $1/\tau$ (curvature, self-avoiding)
- volume V



Let Φ be a random MxN orthoprojector with

$$M = O\left(\frac{K \log(NV\tau^{-1}\epsilon^{-1}) \log(1/\rho)}{\epsilon^2}\right).$$

Then with probability at least $1-\rho$, the following statement holds:

For every pair $x,y \in F$,

$$(1-\epsilon)\,\|x-y\|_2 \le \|\Phi x - \Phi y\|_2 \le (1+\epsilon)\,\|x-y\|_2\,.$$

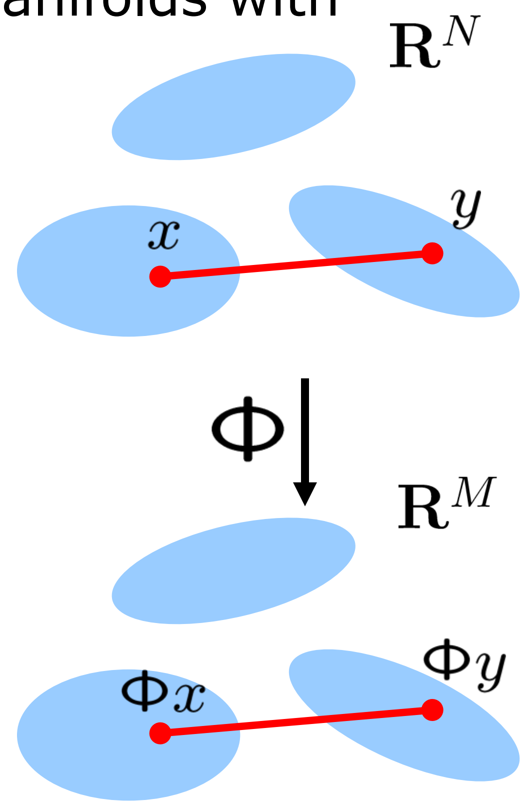
[Baraniuk-Wakin 06]

Multiple Manifold Embedding

Corollary:

Let $M_1, \ldots, M_P \subset \mathbb{R}^N$ be compact K-dimensional manifolds with

- condition number $1/\tau$ (curvature, self-avoiding)
- volume V
- min dist$(M_j, M_k) > \tau$



Let Φ be a random MxN orthoprojector with

$$M = O\left(\frac{K \log(NPV\tau^{-1}\epsilon^{-1}) \log(1/\rho)}{\epsilon^2}\right).$$

Then with probability at least $1-\rho$, the following statement holds:

For every pair $x, y \in \cup M_j$,

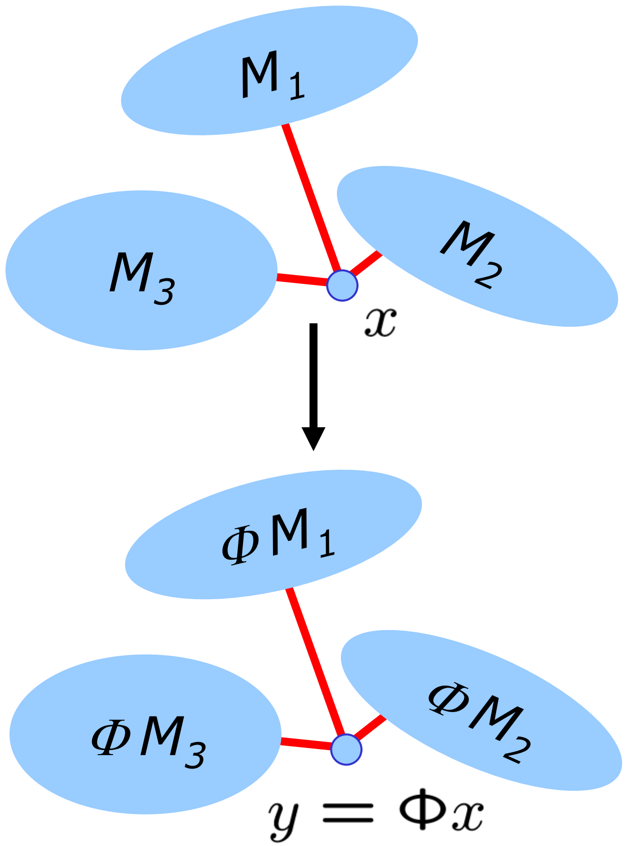
$$(1-\epsilon)\|x-y\|_2 \leq \|\Phi x - \Phi y\|_2 \leq (1+\epsilon)\|x-y\|_2.$$

The *Smashed Filter*

- *Compressive* manifold classification with GLRT
  - nearest-manifold classifier
  - manifolds classified are now $\Phi M_j = \{\Phi f_j(\theta_j) \ : \ \theta_j \in \Theta_j\}$

$$H_j \ : \ y = \Phi(m_j + n), \quad m_j \in M_j$$

$$m_j = f_j(\theta_j)$$

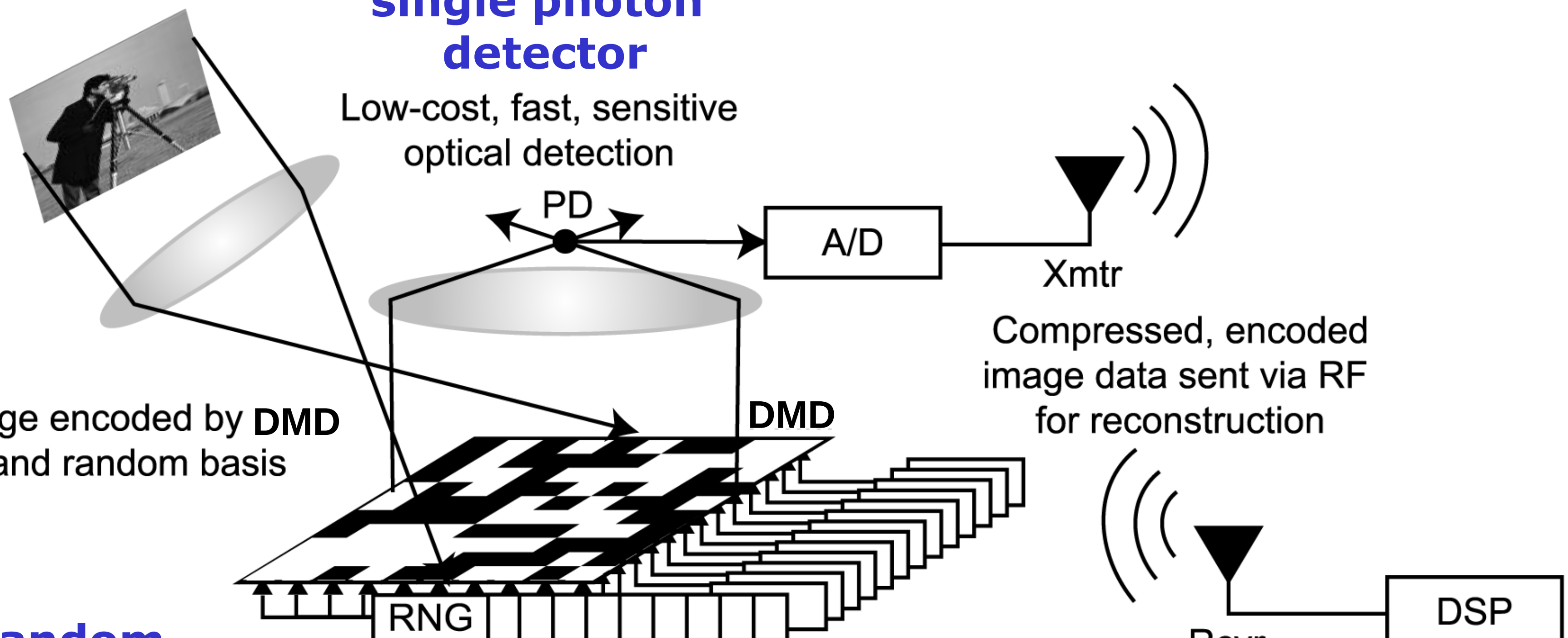


$$\arg\min_{j=1,\ldots,P} \|y - \Phi f_j(\widehat{\theta}_j)\|_2$$

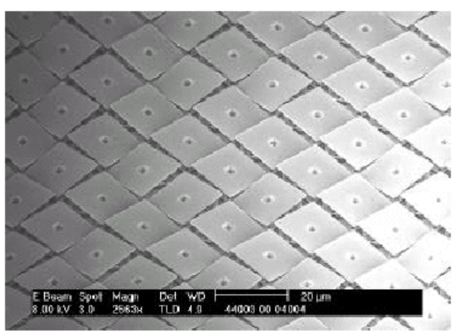
$$\widehat{\theta}_j = \arg\min_{\theta \in \Theta_j} \|y - \Phi f_j(\theta_j)\|_2$$

Rice Single-Pixel Camera

single photon detector



random pattern on DMD array



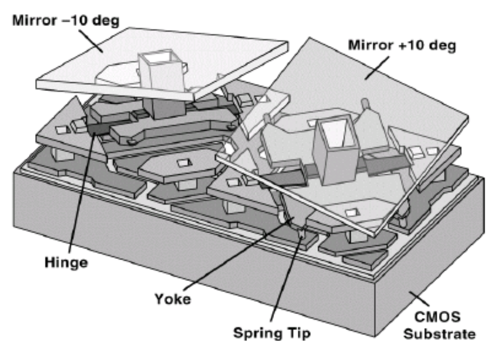


image reconstruction or processing

Image Acquisition

8x sub-Nyquist

Single-Pixel Camera in the News

- Favorite Slashdot comments:

oops, crash, seven million years bad luck !?!

Bet it'd suck to have a bad pixel with that camera, huh? :-)

This is me skydiving
.
This is me swimming with dolphins
.
This is me at the grand canyon
.

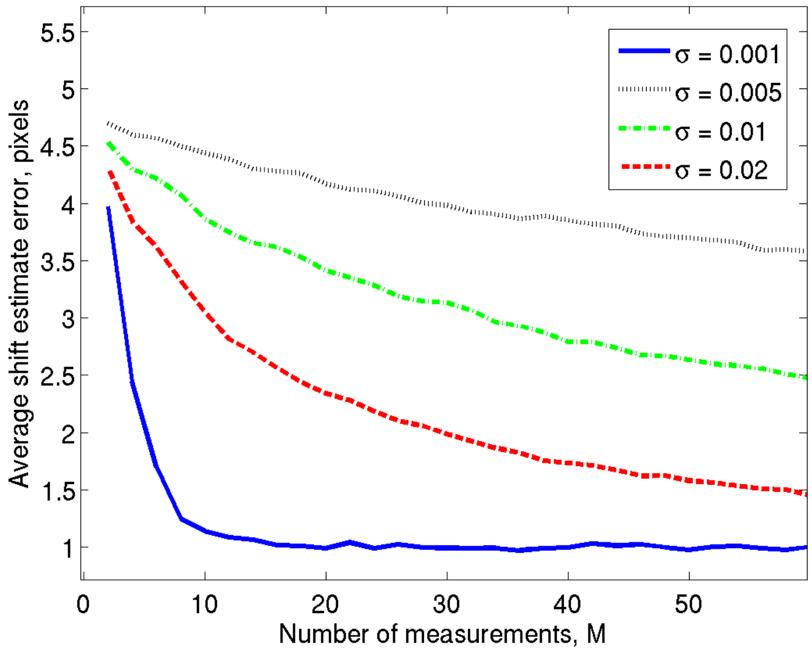
Smashed Filter - Experiments

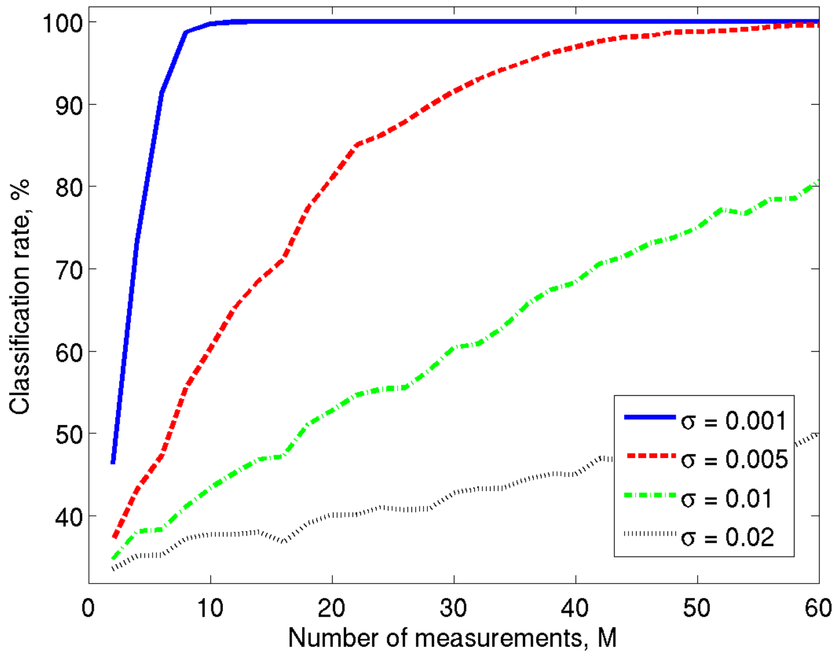
- 3 image classes
  - T-72 tank
  - schoolbus
  - SUV
- Imaged using single-pixel camera with
  - unknown shift
  - unknown rotation



Smashed Filter – Unknown Position

- Image shifted at random, noise added to measurements
  - identify most likely position for each image class
  - identify most likely class using nearest-neighbor test



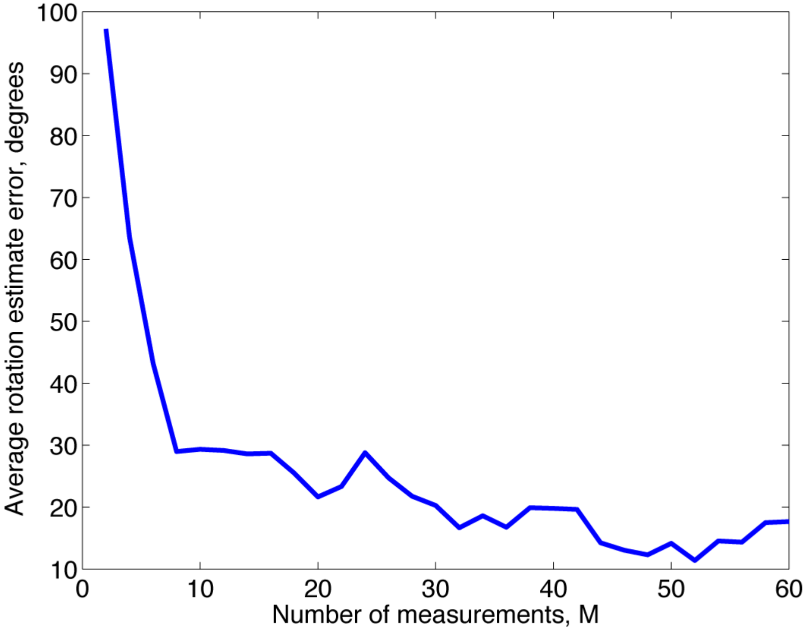


Smashed Filter – Unknown Rotation

- Training set constructed for each class with compressive measurements
  - rotations at 10°, 20°, …, 360°
  - identify most likely rotation for each image class
  - identify most likely class using nearest-neighbor test

- *Perfect* classification with as few as 6 measurements

- Good estimates of the viewing angle with under 10 measurements



Conclusions

- *Smashed filter*
  - efficiently exploits compressive measurements
  - broadly applicable
  - exploits known and unknown manifold structure
  - effective for image classification when combined with single-pixel camera

- Current work:
  - efficient parameter estimation through Newton's method
  - noise analysis
  - compressive k-NN, SVMs, etc.