

Compressive Sensing

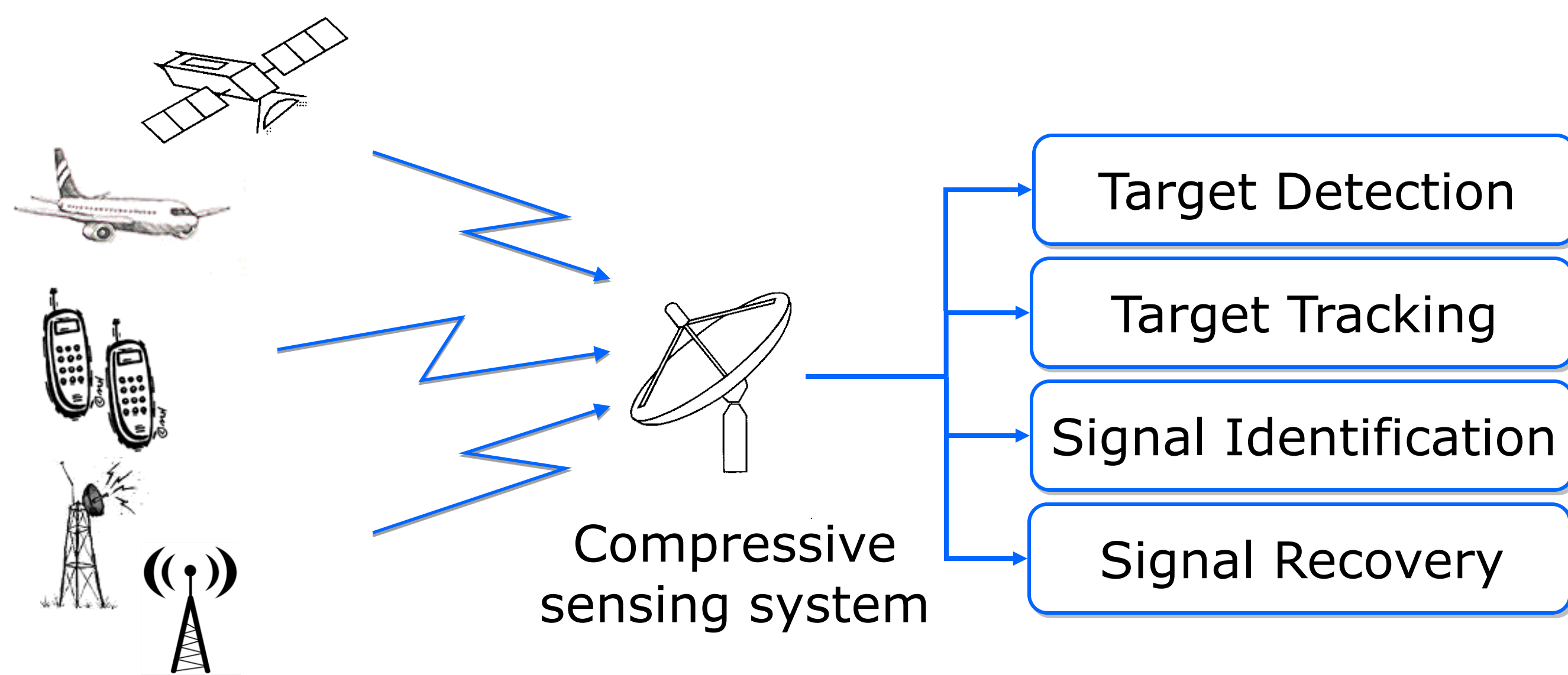
Directly acquire a reduced set of low-dimensional **compressive measurements**

$$\begin{array}{c}
 y \\
 \color{green}{\downarrow} \\
 M \times 1 \\
 \text{measurements}
 \end{array}
 =
 \begin{array}{c}
 \Phi \\
 \color{red}{\downarrow} \\
 M \times N
 \end{array}
 \begin{array}{c}
 \Psi \\
 \color{red}{\downarrow} \\
 N \times N
 \end{array}
 \begin{array}{c}
 \alpha \\
 \color{blue}{\downarrow} \\
 N \times 1 \text{ signal} \\
 K \text{ nonzero} \\
 \text{coefficients}
 \end{array}$$

The **restricted isometry property** (RIP) ensures that Φ captures the information in the signal

$$a\|\alpha\|_2^2 \leq \|\Phi\Psi\alpha\|_2^2 \leq b\|\alpha\|_2^2 \quad \forall \alpha \quad \|\alpha\|_0 \leq K$$

Random Φ satisfy the RIP provided that $M = O(K \log(N/K))$, and provide **information scalability**



Interference Cancellation

Measurements are often contaminated with **interference**

$$y = \Phi x_S + \Phi x_I$$

Seek to remove contribution of x_I to y without necessarily reconstructing $x = x_S + x_I$

x_I might represent actual interference, or signals we wish to ignore for some intermediate processing

General Technique

Assume $x_S \in \mathcal{X}_S$ and $x_I \in \mathcal{X}_I$, where $\langle x_I, x_S \rangle = 0$ for all $x_S \in \mathcal{X}_S, x_I \in \mathcal{X}_I$

Design $\tilde{M} \times M$ matrix P such that

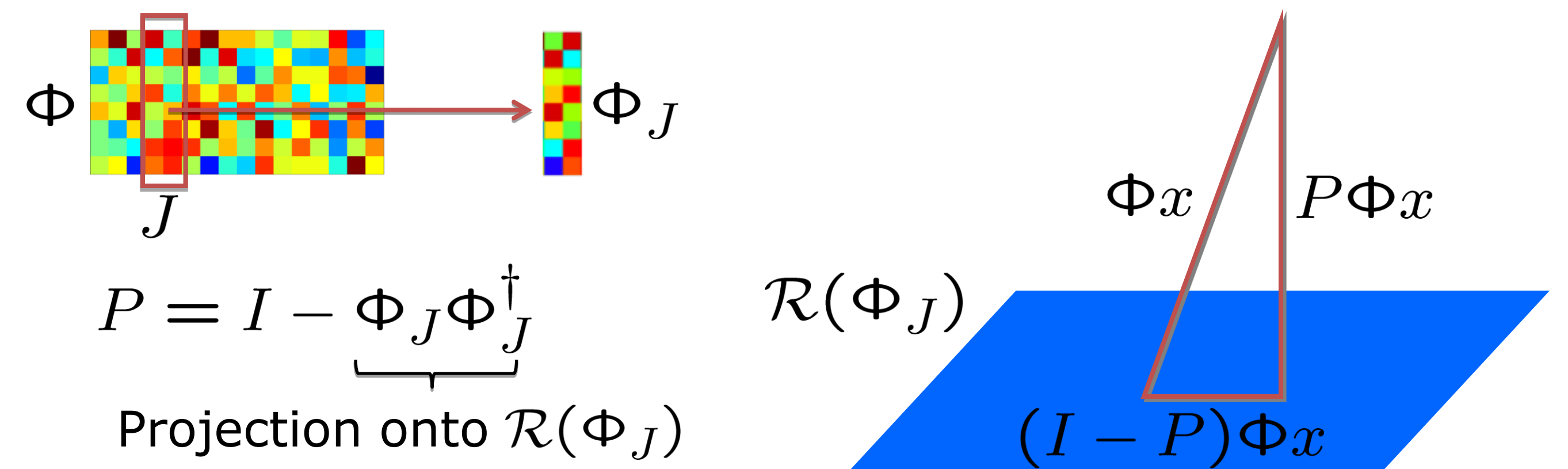
$$\|P(\Phi x_I)\|_2 \approx 0 \quad \text{and} \quad \|P(\Phi x_S)\|_2 \approx \|\Phi x_S\|_2$$

Note: Not always possible

Depends on structure of \mathcal{X}_S and \mathcal{X}_I

Subspace Cancellation

Example: x_I has known support set J of size K_I
Seek P such that $\mathcal{R}(\Phi_J) \subseteq \mathcal{N}(P)$



Observe that $P y = P \Phi x_S + \cancel{P \Phi x_I} = P \Phi x_S$

Theorem: If Φ satisfies the RIP of order $2K_S + K_I$, then $P\Phi$ satisfies

$$(a - (b - a)^2 / 4a) \|x\|_2^2 \leq \|P\Phi x\|_2^2 \leq b \|x\|_2^2$$

for all x such that $\|x\|_0 \leq 2K_S$ and $\text{supp}(x) \cap J = \emptyset$.

Proof exploits two facts:

$$\|\Phi x\|_2^2 = \|P\Phi x\|_2^2 + \|(I - P)\Phi x\|_2^2$$

$$\frac{\|(I - P)\Phi x\|_2}{\|\Phi x\|_2} = \frac{\langle (I - P)\Phi x, \Phi x \rangle}{\|(I - P)\Phi x\|_2 \|\Phi x\|_2} \leq \frac{b - a}{2a}$$

Experiments

Compare four approaches to cancelling interference with known support

1. Cancel-then-recover
2. Modified recovery
3. Recover-then-cancel
4. Oracle-based recover-then-cancel

Compressive domain interference classification is **faster** and **more accurate** than the recover-then-cancel approaches, and compares favorably to the performance of the oracle

