Reconstruction and Cancellation of Sampled Multiband Signals Using Discrete Prolate Spheroidal Sequences

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***Abstract*—There remains a significant gap between the discrete, finite-dimensional compressive sensing (CS) framework and the problem of acquiring a continuous-time signal. In this talk, we will discuss how sparse representations for multiband signals can be incorporated into the CS framework through the use of Discrete Prolate Spheroidal Sequences (DPSS's). DPSS's form a highly efficient basis for sampled bandlimited functions; by modulating and merging DPSS bases, one obtains a sparse representation for sampled multiband signals. We will discuss the use of DPSS bases for both signal recovery and the cancellation of strong narrowband interferers from compressive samples.**

Extended Abstract

In many respects, the core theory of compressive sensing (CS) is now well-settled. Given a suitable number of compressive measurements $y = \Phi x$ of a finite-dimensional vector x, one can recover x exactly if x can be expressed in some dictionary Ψ as $x = \Psi\alpha$ where α is exactly sparse. If α is not exactly sparse, then one can recover an approximation to x, and there exist provably efficient and robust algorithms for performing this recovery.

However, although one of the primary motivations for CS is to simplify the way that high-bandwidth signals are *sampled*, there remains a significant gap between the discrete, finite CS framework and the problem of acquiring a continuous-time signal. Previous work has attempted to bridge this gap by employing two very different strategies. First, in [11] the authors operate directly within the CS framework by employing the simple (but somewhat unrealistic) assumption that the analog signal being sampled is comprised of a sparse linear combination of pure tones with frequencies restricted a harmonic grid. The advantage of this assumption is that it ensures a finite-dimensional sparse representation for x if one chooses Ψ to be the DFT basis. Alternatively, other authors have considered a more realistic signal model—the class of *multiband signals* built from sums of narrowband, bandpass signals—but have performed their analysis largely outside of the standard CS framework [4, 8].

In this talk, we will discuss how sparse representations for multiband signals can be incorporated directly into the CS framework through the use of Discrete Prolate Spheroidal Sequences (DPSS's) [10]. First introduced by Slepian in 1978, the DPSS's can be viewed (and derived) as the discrete-time, finite-length sequences whose Discrete-Time Fourier Transform (DTFT) is most concentrated within a given bandwidth. Most significantly, one can show that for a given sequence of length N and bandlimit $W \in (0, \frac{1}{2})$, the first $\approx 2NW$ DPSS functions form a basis that will capture virtually all of the energy in any length-N sample vector arising from the uniform sampling of a *bandlimited* analog signal. We will expand upon this fact in our talk and explain how, by modulating DPSS's from the baseband to a carrier frequency f_c, one obtains a basis for sample vectors arising from the uniform sampling of *bandpass* analog signals. Merging collections of modulated DPSS's, one then obtains bases for sample vectors arising from the uniform sampling of *multiband* analog signals.

We will discuss the role that such DPSS bases can have in CS. One natural application is in the recovery of windows of multiband signals from the sort of compressive measurements that arise in nonuniform sampling [1] or random demodulation [7] CS architectures. The DPSS bases enjoy a tremendous advantage over the DFT for this purpose; while the DFT representation for a multiband signal is not sparse (it is not even compressible!), the DPSS representation for a multiband signal is almost perfectly sparse and indeed reflects the fundamental information level. We will discuss ongoing work in developing DPSS-based recovery algorithms for CS. Our work on this front differs from [5, 6, 9] in that we consider discrete-time vectors that arise from sampling analog signals with arbitrary multiband spectra.

A second application of the DPSS bases in compressive signal processing involves the cancellation of strong narrowband interferers from a set of compressive samples. Building on the work in [2, 3], we will explain how such interferers can easily be cancelled by orthogonalizing a measurement vector against the DPSS subspace, and we will demonstrate that various signal inference problems can be solved with a high degree of accuracy after the cancellation of an interferer many times stronger than the signal itself.

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