ECE 4803, Fall 2020

Homework #5

Due Tuesday, October 6, at 9:30am

1. Prepare a one paragraph summary of what we talked about in class since the last assignment. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other classes?). The more insight you give, the better.

2. Show that the following functions are convex. For each, indicate if it is strictly convex or not. [Hint: recall the second-order conditions of convexity from the notes.]

   (a) $f(x) = x^2$

   (b) $f(x) = e^{x^2}$

   (c) $f(x) = \log(1 + e^x)$

3. Consider $f(\boldsymbol{x}) = \|\boldsymbol{x}\|$, where $\|\boldsymbol{x}\|$ denotes any valid norm.

   (a) Prove that $f(\boldsymbol{x})$ is convex using the basic properties of a norm.

   (b) Give an example of a norm that is strictly convex.

   (c) Give an example of a norm that is *not* strictly convex.

4. (a) Consider the so-called "rectified linear unit" or ReLU activation function that is commonly used in neural networks:

   $$r(x) = \max(0, x).$$

   Show that $r(x)$ is convex.

   (b) Let $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$ be convex functions on $\mathbb{R}^N$. Generalize the previous result by showing that

   $$f(\boldsymbol{x}) = \max\{f_1(\boldsymbol{x}), f_2(\boldsymbol{x})\}$$

   is convex.

   (c) If $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$ are convex, can you say anything about the convexity or concavity of

   $$f(\boldsymbol{x}) = \min\{f_1(\boldsymbol{x}), f_2(\boldsymbol{x})\}?$$

   Sketch a one-dimensional example that supports your argument.

Last updated 17:19, October 19, 2020