ECE 3803
Optimization for Information Systems
Fall 2021

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Practicalities

Review the syllabus at your leisure

I expect everyone to behave responsibly

Format/time of office hours TBD

Small ***anonymous*** survey

Prerequisites

- Linear algebra
  - matrices, vectors, eigenvalues, eigenvectors, …
- Multivariable calculus
  - partial derivatives, gradients, …
- Python (or MATLAB) programming experience
  - we will use Python, but this should be easy to learn if you have MATLAB experience

Course materials

There is no required textbook for this course

A list of other useful books and links to relevant papers will be posted on the course webpage

Lecture notes will also be posted on the course webpage

I plan to make extensive use of Piazza

Grading

- Homework (40%)
  - ≈ 10 assignments, each graded out of 100
  - if N assignments, homework score will be out of (N-1)·100
- Quizzes (30%)
  - tentatively scheduled for September 21 and November 2
- Final exam (25%)
  - December 16 (8-10:50am)
- Participation (5%)

Optimization for Information Systems

Introduction to the fundamentals of optimization

Focus on algorithms and applications involving systems that manipulate information

- filters
- tracking/localization
- digital communication
- control systems
- machine learning
- robotics/autonomous systems
- …

We typically want our systems to be ***optimal***

What is optimization?

Given a collection of elements,
select the “best” one

Optimization in Antiquity

Between 29 and 19 BC,
Virgil wrote the Aeneid

The Aeneid tells of the Tyrian princess Dido, who founded Carthage
(circa 1000 BC)

Dido is supposed to have asked the local Berber king for a plot of land limited to that which could be enclosed with the hide of a bull

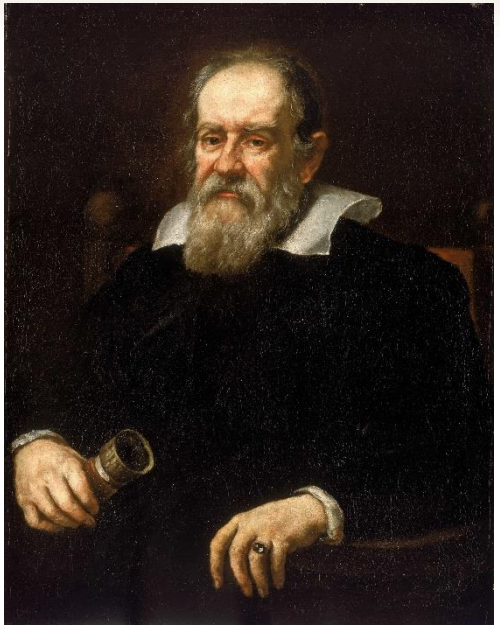
Isoperimetric problem

Among all shapes of a given perimeter,
which one encloses the greatest area?

First known proof that the answer is a circle reportedly due to the Greek mathematician Zenodorus (circa 200 BC)

A more modern example

Galileo Galilei (1564-1642)

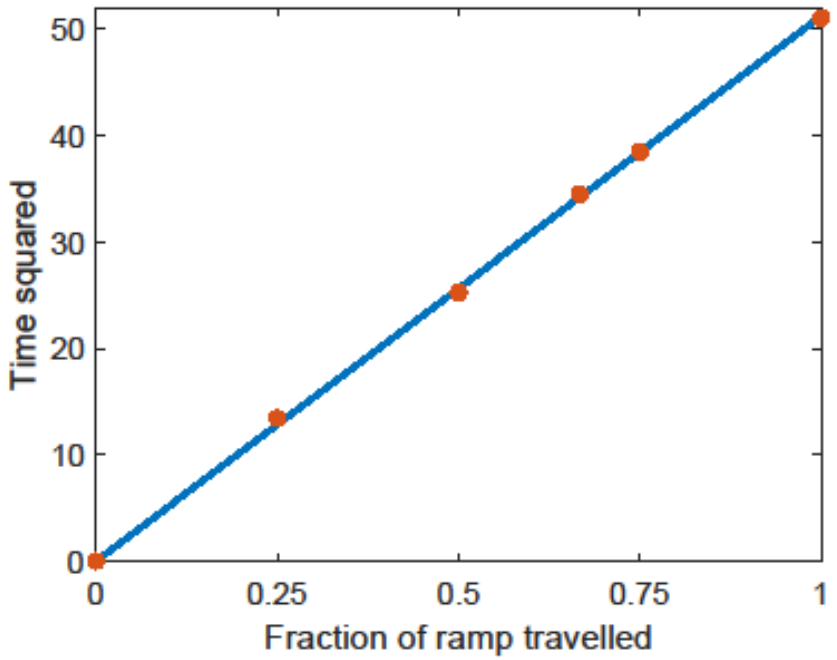


An object in free-fall (no friction) will experience uniform acceleration

Distance travelled should be proportional to the square of the elapsed time

A more modern example

A more modern example



Hanging chain (catenary) problem

What is the shape of the curve assumed by a loose string (or chain) hung freely from two fixed points?

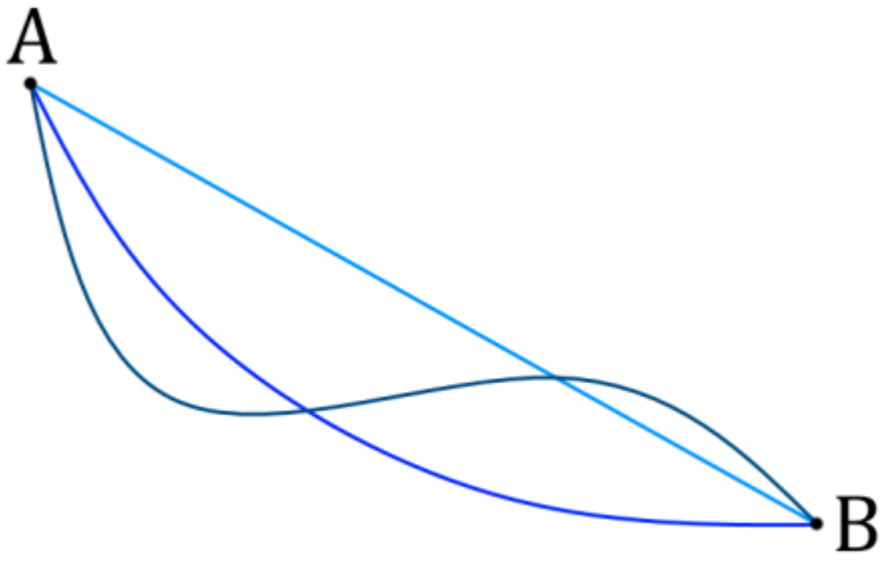
What is the shape of a chain (of fixed length) between two fixed points that minimizes the potential energy of the chain?

Galileo conjectured that the answer would be a parabola… but this is not quite right (turns out to be a hyperbolic cosine)

Hanging chain (catenary) problem

Brachistochrone problem

What is the shape of the curve, with given endpoints, down which a particle would slide (under gravity but ignoring friction) in minimal time?

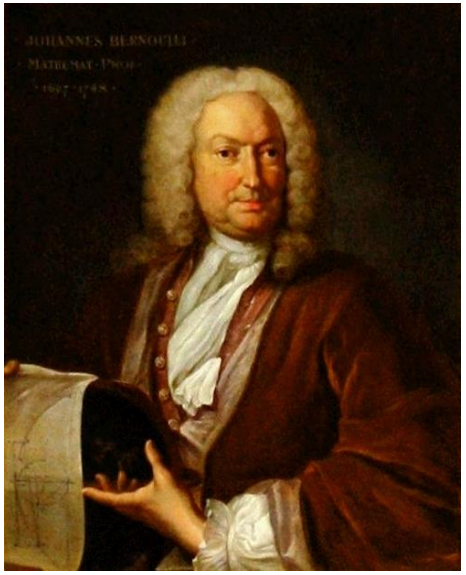


Galileo conjectured that the answer would be a circular arc

Wrong again, Galileo!

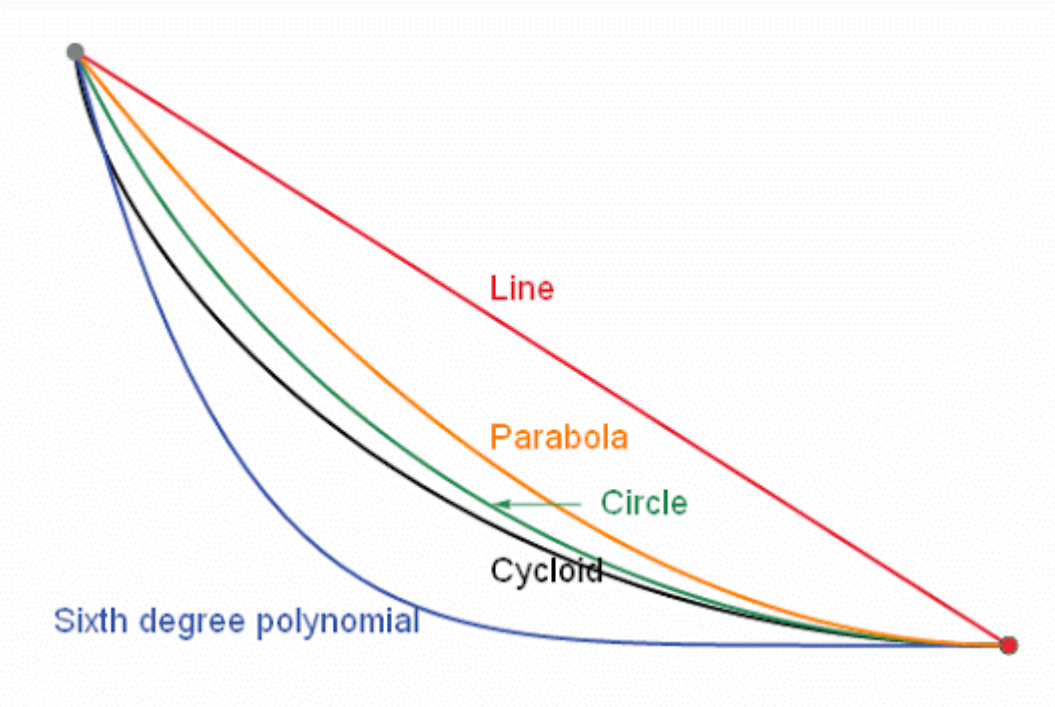
These problems were notoriously challenging

"Galileo, perhaps the most clever of us all, was at a disadvantage in dealing with these problems because he did not have at his disposal the mathematical tools of the calculus" – Johann Bernoulli



Brachistochrone problem published by Johann in 1696 as a challenge to the mathematical community

(The answer is a cycloid)

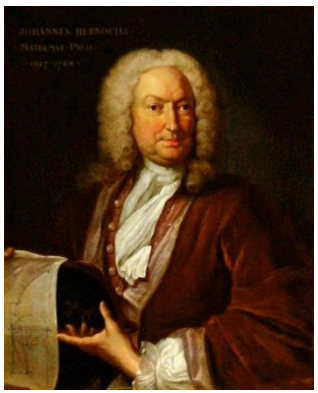


Who is worthy of praise?

Several mathematicians eventually offered solutions

- Johann
- Jakob Bernoulli (Johann's older brother)
- Gottfried Wilhelm Leibniz (Jakob's advisor)
- Guillaume Antoine, Marquis de L'Hôpital (Johann's student)
- Isaac Newton (not a friend of the Bernoulli family)

Mäthy Crüe



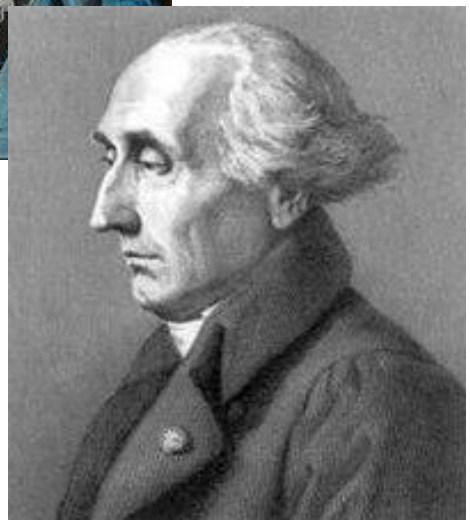


Beyond the ball and chain…

These optimization problems were incredibly influential on the development of modern mathematics

Motivation of future work by Euler

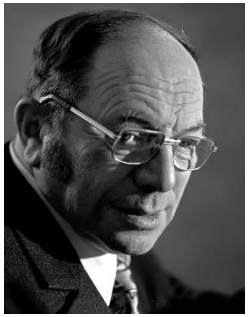
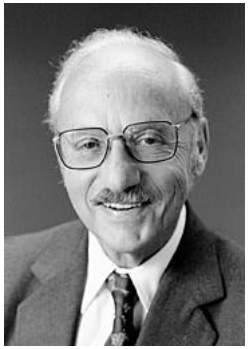
Which in turn inspired Lagrange
to lay the systematic foundations
for the theory of optimization
that we will learn about in this class



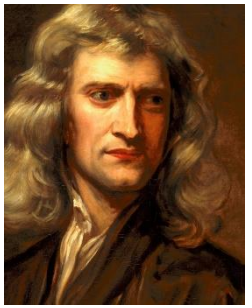
Optimization in the 20th Century

optimize using calculus → optimize using computers

- Linear programming (1940s)


- Nonlinear optimization
  - gradient descent
  - Newton's method


- Constrained optimization
  - Karush-Kuhn-Tucker conditions

Optimization in the 21st Century

- Accelerated gradient descent
- Nonsmooth optimization
- Stochastic gradient descent
- Parallel/decentralized/large scale optimization

- Applications in
  - signal processing
  - control systems
  - machine learning
  - robotics

What is optimization?

Given a collection of elements,
select the “best” one

What is optimization?

$$\underset{\boldsymbol{x}}{\text{minimize}} \; f(\boldsymbol{x}) \;\; \text{subject to} \;\; \boldsymbol{x} \in \mathcal{X}$$

decision variables

objective function

constraint set

We seek an $\widehat{\boldsymbol{x}} \in \mathcal{X}$ such that $f(\widehat{\boldsymbol{x}}) \leq f(\boldsymbol{x})$ for all $\boldsymbol{x} \in \mathcal{X}$

We call such an $\widehat{\boldsymbol{x}}$ a ***minimizer*** of f in $\mathcal{X}$, and a ***solution*** to our optimization problem

Fundamental questions

- Does a solution exist?
- Is there a unique solution?
- Can we easily check if we have found a solution?
- Can we easily find a solution?