SCALABLE INFERENCE AND RECOVERY FROM COMPRESSIVE MEASUREMENTS

Mark A. Davenport, Richard G. Baraniuk
Rice University
Department of Electrical and Computer Engineering

Michael B. Wakin
California Institute of Technology
Department of Applied and Computational Mathematics

Overview

Data deluge carries technological challenges:

- acquire, store, and process huge amounts of high-dimensional data

Reduce these burdens by reducing the dimensionality of the data

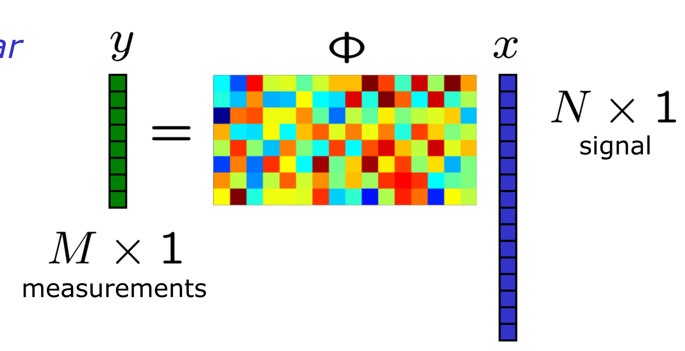
- capture key signal information in a reduced set of measurements
- later recover signal or statistics of interest

Why is dimensionality reduction even possible?

- signals and data may have *low-dimensional structure*,
- statistics of interest may involve *low-complexity inference*,
- or both

Several precedents for using *random, nonadaptive, linear measurements*:

- estimating approximate nearest-neighbors
- estimating statistics of large data sets
- compressed sensing



Common link for success in these applications is a *concentration of measure phenomenon*:

- let $x \in R^N$ and let Φ: $R^N \times R^M$ be a random matrix with iid Gaussian entries; then

$$\mathbf{P}(|\,\|\Phi x\|_{\ell_2^M}^2 - \|x\|_{\ell_2^N}^2| \geq \epsilon \|x\|_{\ell_2^N}^2) \leq 2e^{-M\epsilon^2/4}$$

- other distributions for Φ also possible

This property has substantial implications in a number of settings

- compressive *hardware* under development

Low-Complexity Inference

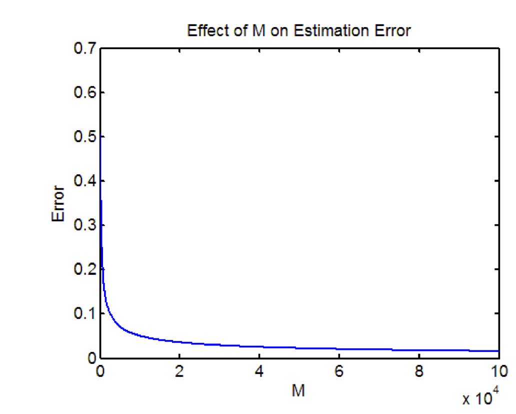
Estimation

Suppose we want to estimate $\langle x, s\rangle$ from $y = \Phi x$

With probability at least $1-\delta$,

$$|\langle \Phi x, \Phi s\rangle - \langle x, s\rangle| \leq \kappa_\delta \frac{\|x\|_2 \|s\|_2}{\sqrt{M}}$$

where $\kappa_\delta = 2\sqrt{12 \log\left(\frac{6}{\delta}\right)}$.



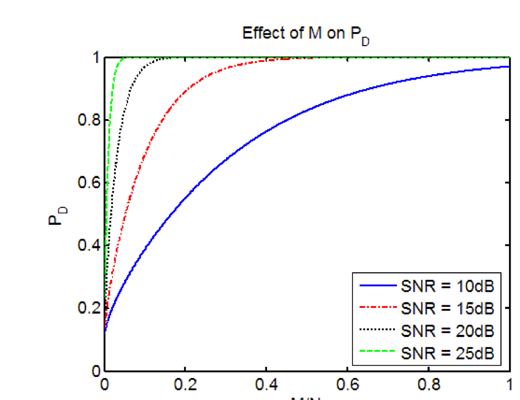
Detection

Suppose we want to determine between

$$H_0 \ : \ y = \Phi n$$
$$H_1 \ : \ y = \Phi(s+n)$$

Neyman-Pearson optimal detector is the *compressive matched filter* $t = \langle y, \Phi s\rangle$, with ROC

$$P_D(\alpha) = Q\left(Q^{-1}(\alpha) - \frac{\|\Phi s\|_2}{\sigma}\right) \approx Q\left(Q^{-1}(\alpha) - \sqrt{\frac{M}{N}}\frac{\|s\|_2}{\sigma}\right)$$

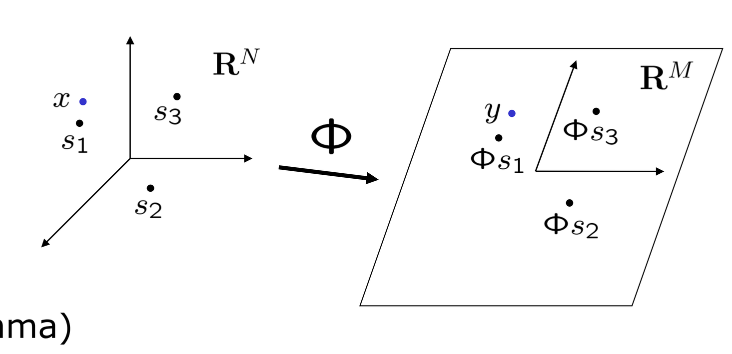


Classification

More generally, suppose we want to distinguish between s_1, s_2, s_3, …

$$\left.\begin{aligned} t_1 &= \|y - \Phi s_1\|_2 \\ t_2 &= \|y - \Phi s_2\|_2 \\ t_3 &= \|y - \Phi s_3\|_2 \end{aligned}\right\}$$

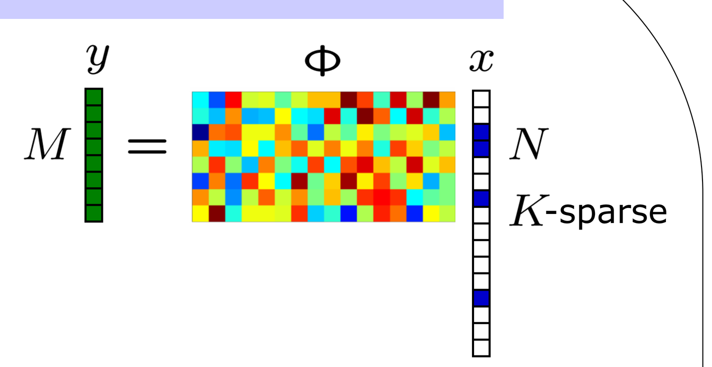
Distances preserved by concentration of measure (Johnson-Lindenstrauss lemma)



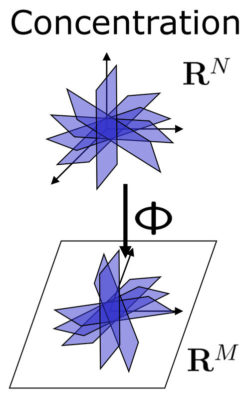
Low-Dimensional Models

Sparse Signal Models

Compressed Sensing takes advantage of the low-dimensional structure of sparse signals



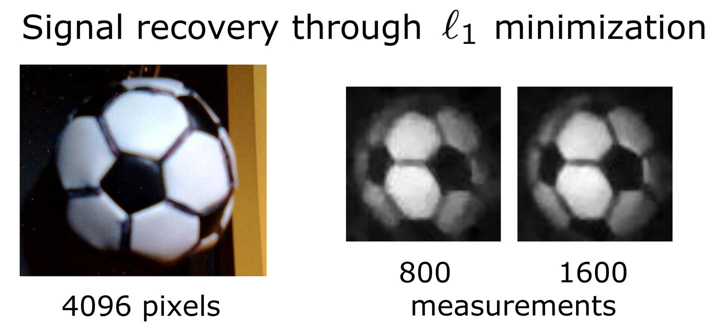
Concentration of measure allows embedding



$$M = O(K \log N)$$

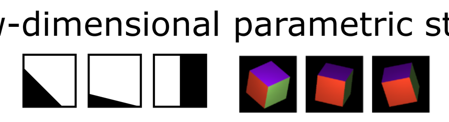
$$(1-\epsilon) \leq \frac{\|\Phi x\|_2}{\|x\|_2} \leq (1+\epsilon)$$

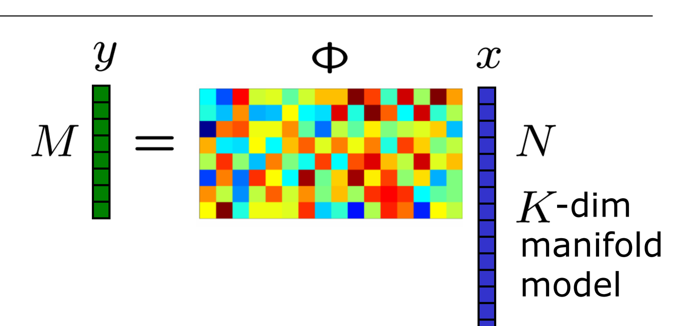
Signal recovery through ℓ_1 minimization



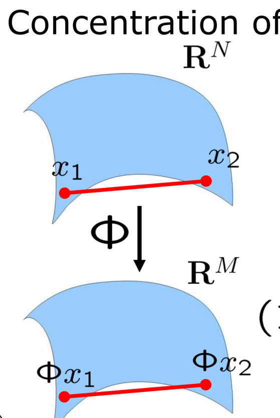
Manifold Signal Models

Low-dimensional parametric structure





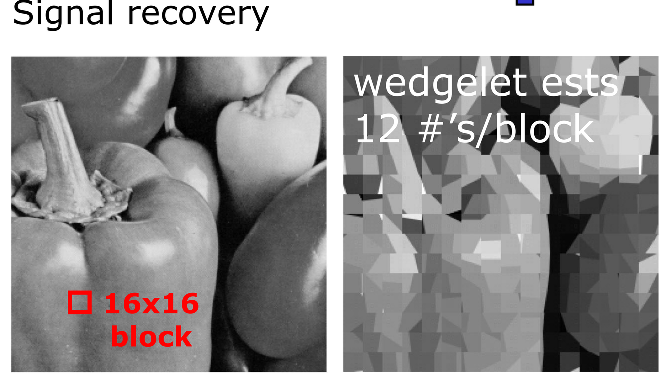
Concentration of measure



$$M = O(K \log N)$$

$$(1-\epsilon) \leq \frac{\|\Phi x_1 - \Phi x_2\|_2}{\|x_1 - x_2\|_2} \leq (1+\epsilon)$$

Signal recovery



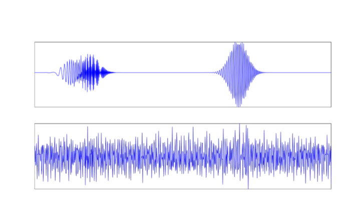
Low-Complexity Inference with Low-Dimensional Models

Estimation, Detection, and Classification

Combine low-complexity inference with sparse models

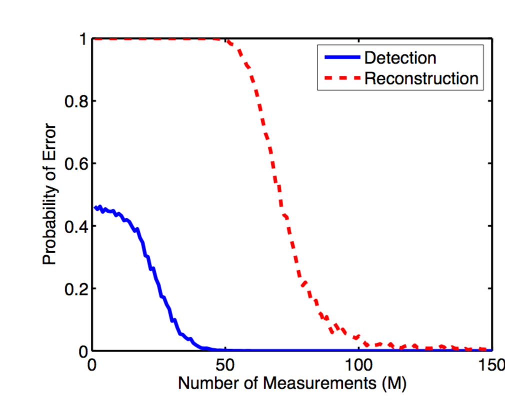
IDEA – Incoherent Estimation and Detection Algorithm

- greedy algorithm for obtaining a partial reconstruction
- estimate and cancel sparse/compressible noise
- threshold recovered coefficients



Case study: Wideband chirps in narrowband noise

- weak signal of interest in heavy, narrowband noise
- can detect using *3x fewer measurements* compared to greedy reconstruction

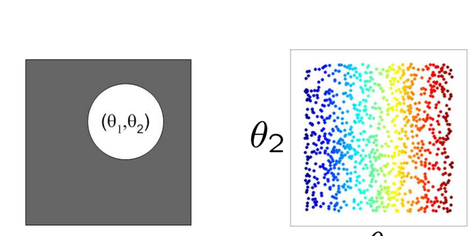


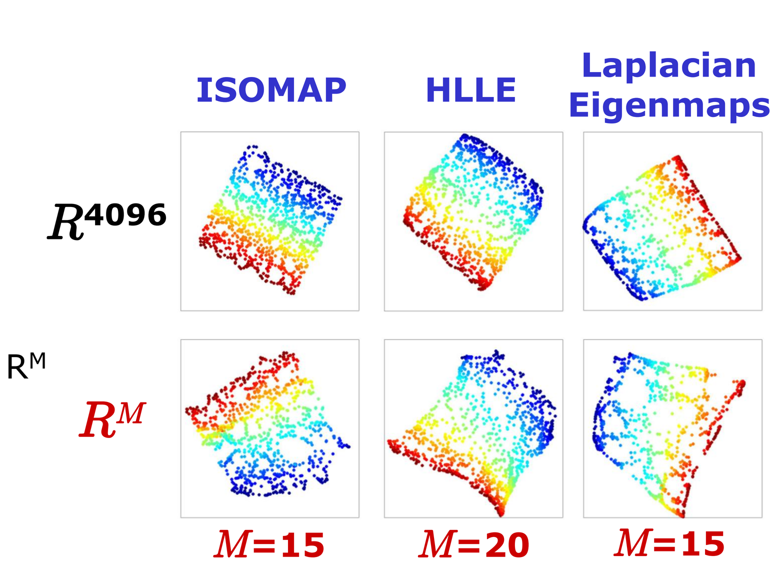
Manifold Learning

Random projections preserve key manifold properties

- distances (ambient & geodesic) between points
- dimension, volume, and topology of the manifold
- lengths and curvature of paths on the manifold

Learn these features from sampled, *projected* data in R^M





dsp.rice.edu/cs

Decreasing Complexity of Signal

Decreasing Complexity of Inference