Joint Manifold Models for Collaborative Inference

Mark A. Davenport, Marco F. Duarte, Chinmay Hegde, Richard G. Baraniuk

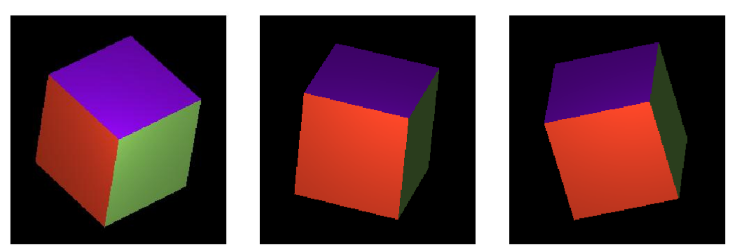
Rice University

Joint Manifold Models

Manifold models

High-dimensional signals often possess low-dimensional geometric structure

Example: $SO(3)$



K-dimensional *parameter* θ captures degrees of freedom in signal $x \in \mathbb{R}^N$

Joint manifolds

In many settings, an ensemble of signals will share a common underlying parameterization

Given submanifolds $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_J \subset \mathbb{R}^N$

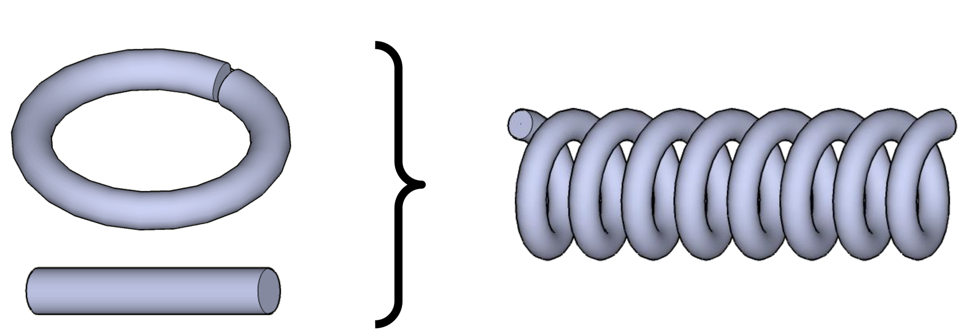
- K-dimensional
- jointly homeomorphic

The joint manifold $\mathcal{M}^* \subset \mathbb{R}^{JN}$ is the *concatenation* of $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_J$

Example:

$$\mathcal{M}_j = \{f_j(\theta), \theta \in \Omega\}$$

$$\mathcal{M}^* = \{f^*(\theta), \theta \in \Omega\} = \{[f_1(\theta); f_2(\theta); \ldots; f_J(\theta)], \theta \in \Omega\}$$



$$f^*(\theta) = \begin{bmatrix} \theta \\ \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Joint manifold inherits

- compactness
- smoothness
- volume: $\max_j V_j \leq V^* \leq \sum_{j=1}^{J} V_j$
- condition number ($1/\tau$): $\frac{1}{\tau^*} \leq \max_j \frac{1}{\tau_j}$

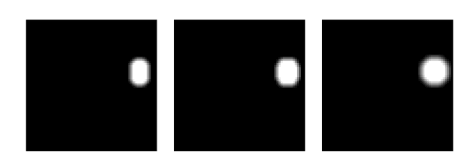
Inference with Joint Manifolds

Joint manifold learning

The joint manifold can be *well-conditioned* even when the component manifolds are *ill-conditioned*

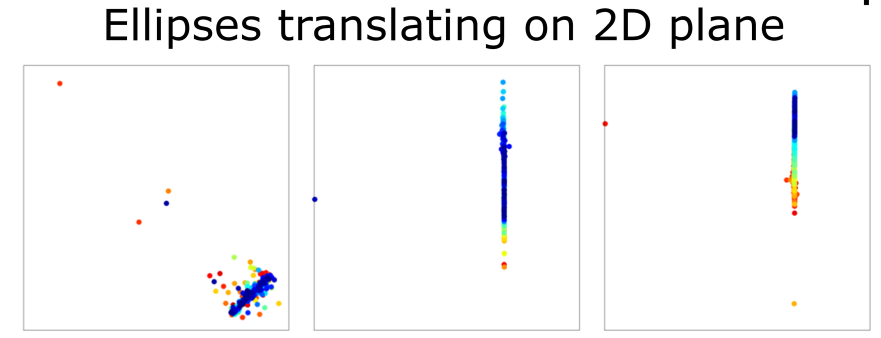
- ***better performance with fewer samples***
- ***increased tolerance to noise***

Example: Manifold learning

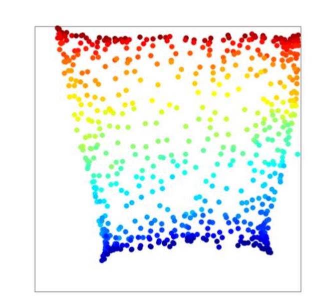


Ellipses translating on 2D plane

Find 2D embedding of a dataset of noisy, high-dimensional signals



Embeddings learned separately



Joint embedding

Dimensionality – curse or blessing?

By increasing the dimensionality we are more easily able to identify structure and ignore noise

Drawback:

Computational demands can be overwhelming

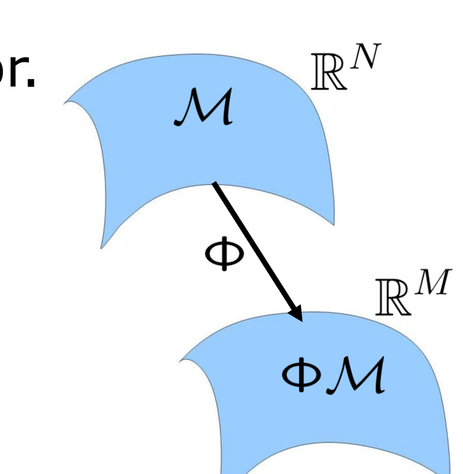
The random projection method

Let Φ be an $M \times N$ random orthoprojector.

Let $\mathcal{M}$ be a compact, K-dimensional, Riemannian submanifold in $\mathbb{R}^N$.

If

$$M = O\left(\frac{K \log(NV\tau^{-1}\epsilon^{-1}) \log(1/\rho)}{\epsilon^2}\right).$$



then with probability at least $1 - \rho$

$$(1-\epsilon)\,\|x-y\|_2 \leq \|\Phi x - \Phi y\|_2 \leq (1+\epsilon)\,\|x-y\|_2$$

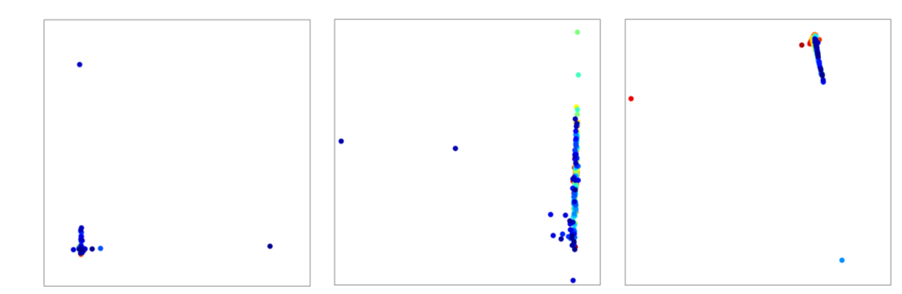
for all $x, y \in \mathcal{M}$. [Baraniuk-Wakin, 2006]

dsp.rice.edu

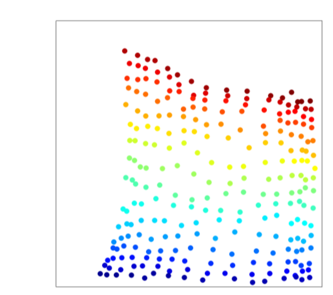
Collaborative Inference

Efficient manifold learning

Random projections cause little distortion in the learned embeddings, and allow for the joint embedding of large numbers of sensors



Embeddings learned separately from 100 random projections of 45x45-pixel images

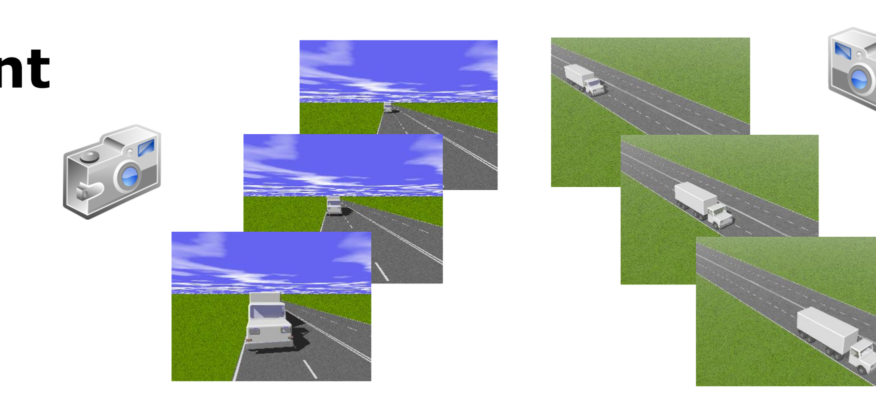


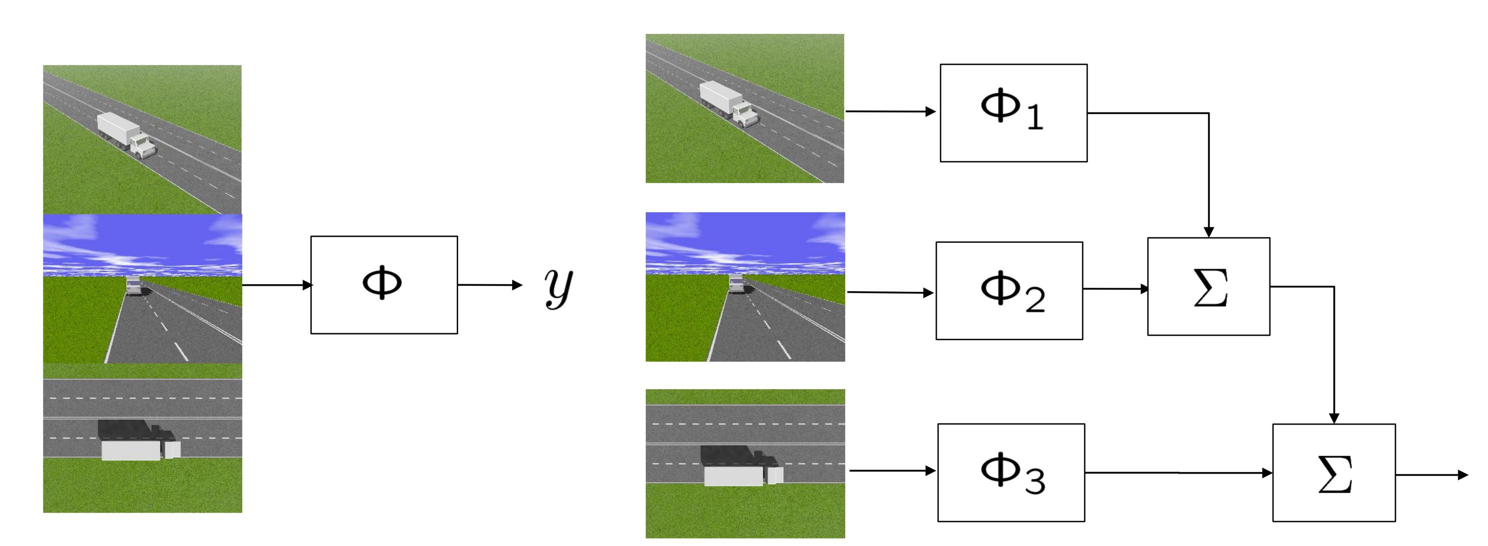
Joint embedding learned from 100 random projections of joint manifold (20 component manifolds)

Distributed classification

We can project data independently at each sensor, adding the measurements to obtain the joint projections

- **simple/efficient data fusion**
- **sensors operate independently**





J=3 CS cameras
N=320x240, M=200

Two classes
1. Truck with cargo
2. Truck with no cargo

Joint manifold approach performs almost as well as the best camera