SPARSE SIGNAL DETECTION FROM INCOHERENT PROJECTIONS

Overview

Compress signals while preserving sufficient statistics

- requires no knowledge of signal structure
- requires no knowledge of type of statistics

Key idea: random projections

- universal measurement scheme for sparse signals

Connection: Compressive Sensing (CS)

- new theory for *recovering* sparse signals from random projections

Information scalability

- generalize CS to recover different levels of information from random projections
- requires fewer measurements, lower complexity

Sensing and Compression

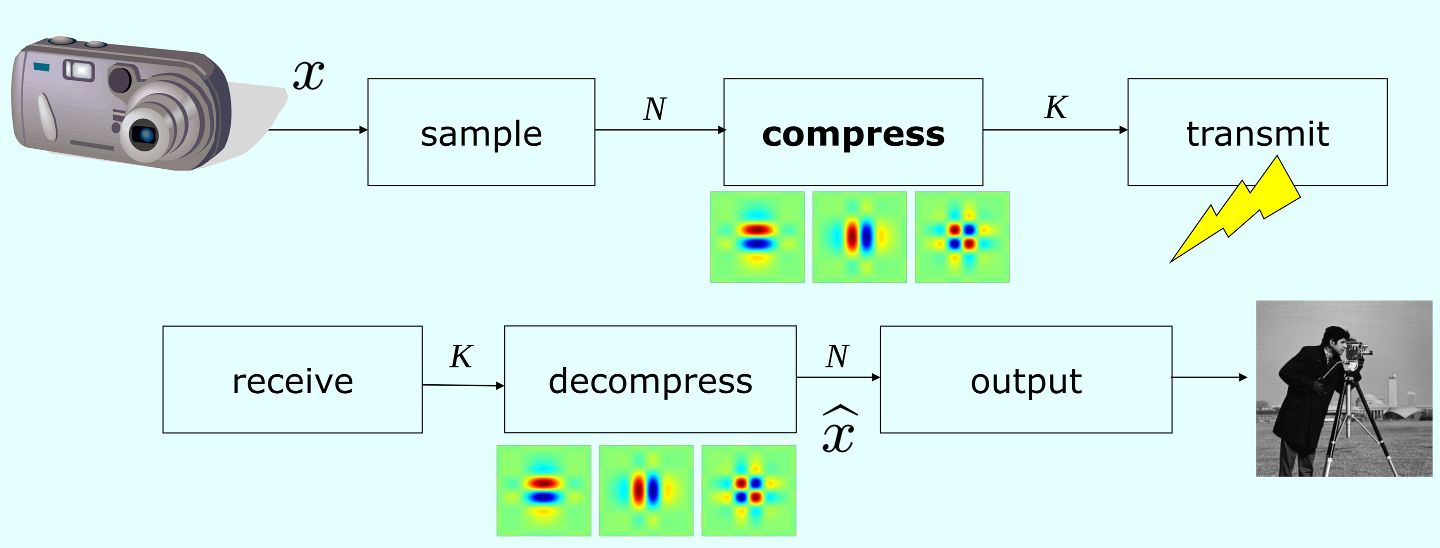
Sparse signal representation

- approximate length-N signal x using K coefficients

$$x = \sum_{i=1}^{N} \theta_i \psi_i \qquad \widehat{x} = \sum_{K \ll N \text{ largest terms}} \theta_i^q \psi_i$$

Conventional sensing

- sample (at Nyquist rate)
- compress (using model such as sparsity)
- throw away most coefficients



Compressive Sensing
[Donoho; Candes, Romberg, Tao]

Measure projections of signal onto incoherent basis

- e.g., Gaussian, Bernoulli +/-1

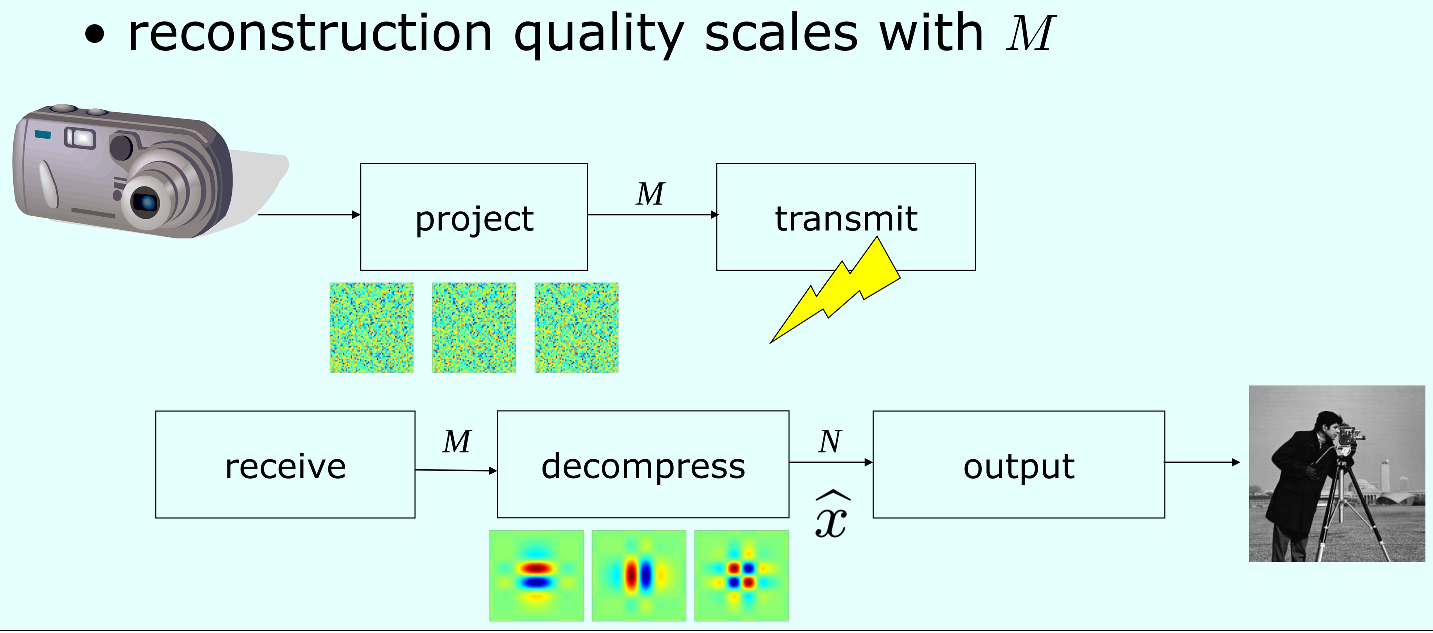
$$y = \Phi x = \Phi \Psi \theta \qquad y: \; M \times 1 \quad \Phi: \; M \times N \quad x: \; N \times 1$$

- mild oversampling: $M \sim K \log N$

Recovery: find sparsest signal x that explains measurements y

Random projections give universal, robust encoding

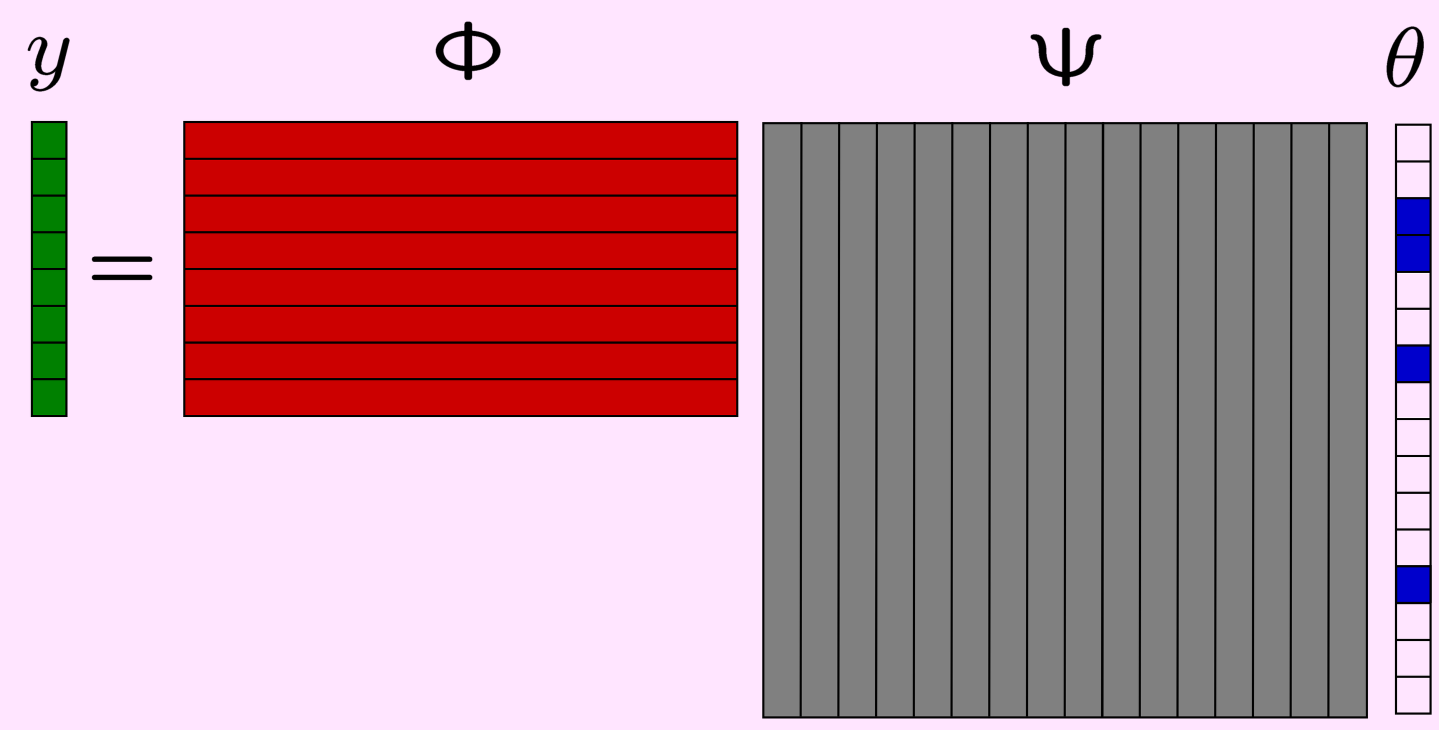
- sparsity basis Ψ known only at *decoder*
- reconstruction quality scales with M



Greedy CS Recovery

Matching Pursuit

- initialize: set residual $y_r = y$
- select column of $\Psi\Phi$ most correlated with y_r
- subtract column from y_r
- iterate until T steps or $||y_r||$ is small

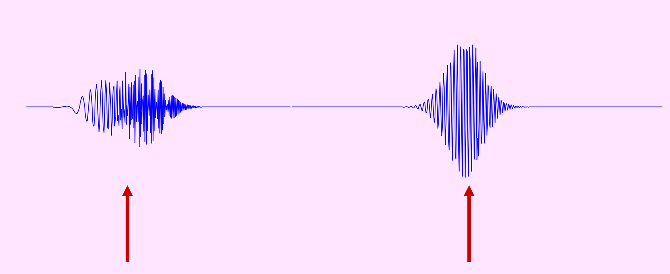


Information Scalability

Different applications require different levels of information:

- signal reconstruction
- signal approximation
- parameter estimation
- signal detection/classification

fewer **measurements;**
lower **computational complexity**



Random projections are *universal* with respect to

- sparsity-inducing basis
- level of information desired about x

Given y, directly estimate sufficient statistics about x

- exploit prior knowledge that x is sparse

Incoherent Detection and Estimation Algorithm (IDEA)

Problem Setup:

- Ω = target indices in dictionary Ψ
- given $y = \Phi x$, decide between

$$\mathcal{H}_0: \quad \theta_\Omega = 0 \quad \text{vs.} \quad \mathcal{H}_1: \quad \theta_\Omega \neq 0$$

If x were provided and Ψ were orthonormal

- matched filtering

If x were provided and Ψ were redundant

- analogous to multiuser detection (NP-hard)
- solution: sparse approximation algorithms

IDEA: adapt Matching Pursuit for compressed detection

- reduce # iterations; increase stopping energy
- check coefficients of θ_Ω to make decision
- detection possible without accurate reconstruction

Marco F. Duarte, Mark A. Davenport,
Michael B. Wakin, and Richard G. Baraniuk

Rice University
Department of Electrical and Computer Engineering

Case Study: Dictionary-Based Detection

Two hypotheses:

$$\mathcal{H}_0: \quad x = n + \omega \quad \text{vs.} \quad \mathcal{H}_1: \quad x = s + n + \omega$$

$$s = \Psi_s \theta_s, \ \|\theta_s\|_0 = K_s$$

$$n = \Psi_n \theta_n, \ \|\theta_n\|_0 = K_n$$

ω : noise

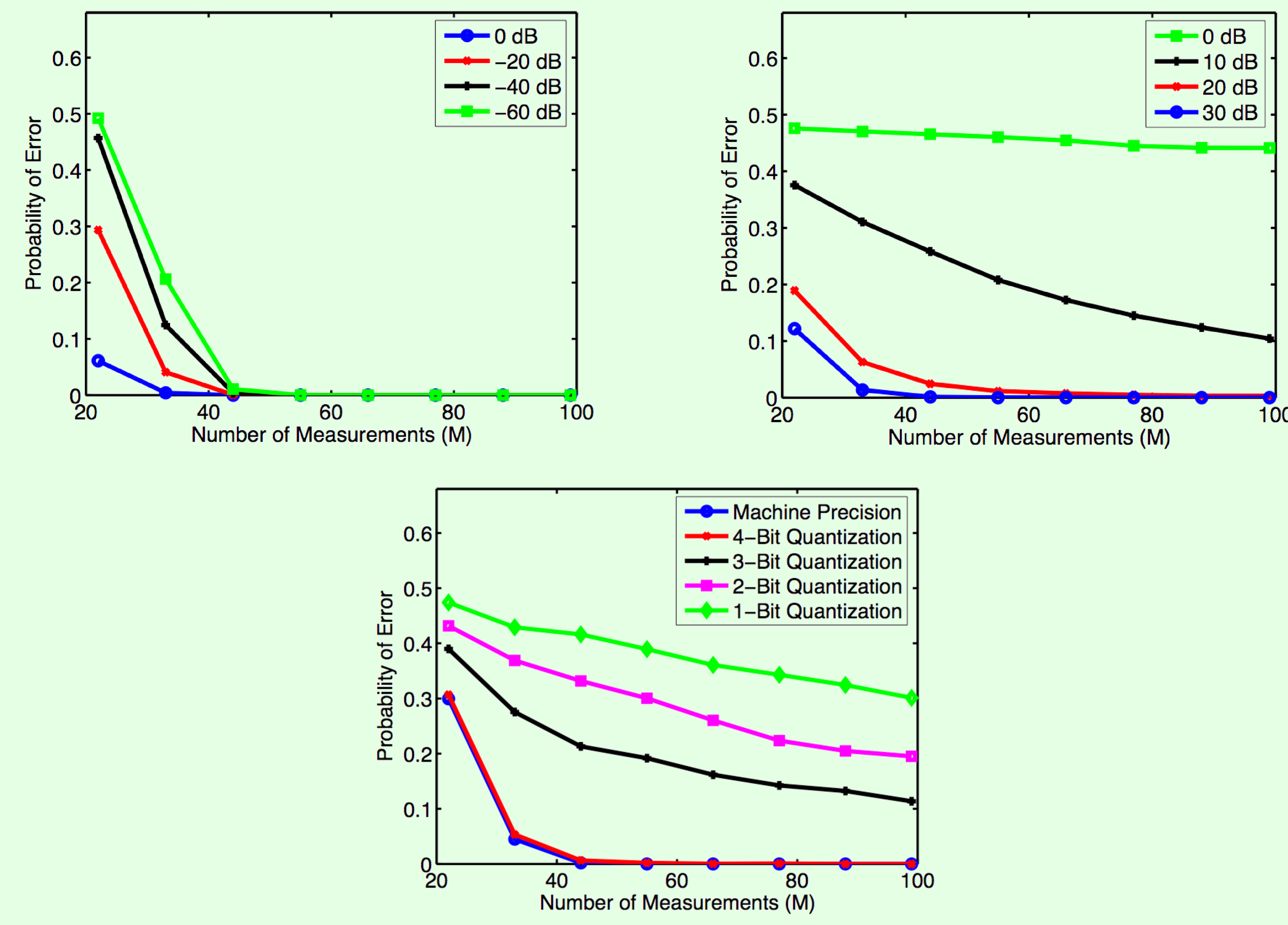
Signal s and interference n components are sparse in different dictionaries

Concatenate dictionaries and restate hypotheses

$$x = [\Psi_s \ \Psi_n] \begin{bmatrix} \theta_s \\ \theta_n \end{bmatrix} + \omega =: \Psi\theta + \omega$$

$$\mathcal{H}_0: \quad \theta_s = 0 \quad \text{vs.} \quad \mathcal{H}_1: \quad \theta_s \neq 0$$

Robustness



IDEA detection robust to

- strength of sinusoidal interference
- noise energy
- quantization of measurements

Wideband Signals in Strong Narrowband Interference

Setup:

- weak wideband chirps
- strong narrowband sinusoidal interference
- noise

Challenge: chirp detection

- chirps too weak for energy detection
- sinusoids may dominate in frequency domain

Solution: IDEA

- chirplet dictionary Ψ_s to sparsify chirps
- Fourier dictionary Ψ_n to sparsify interference

Experiment:

- signal length N = 1024
- chirplet dictionary with 432 distinct length-64 chirps
- sparsities K_s = 5 (when present) and K_n = 6

Extension to Classification

Suppose x has sparsity K in one of C bases/dictionaries

- each basis represents a signal class

Classification problem:

- determine class to which x belongs
- sparse approximation from each dictionary requires all N samples

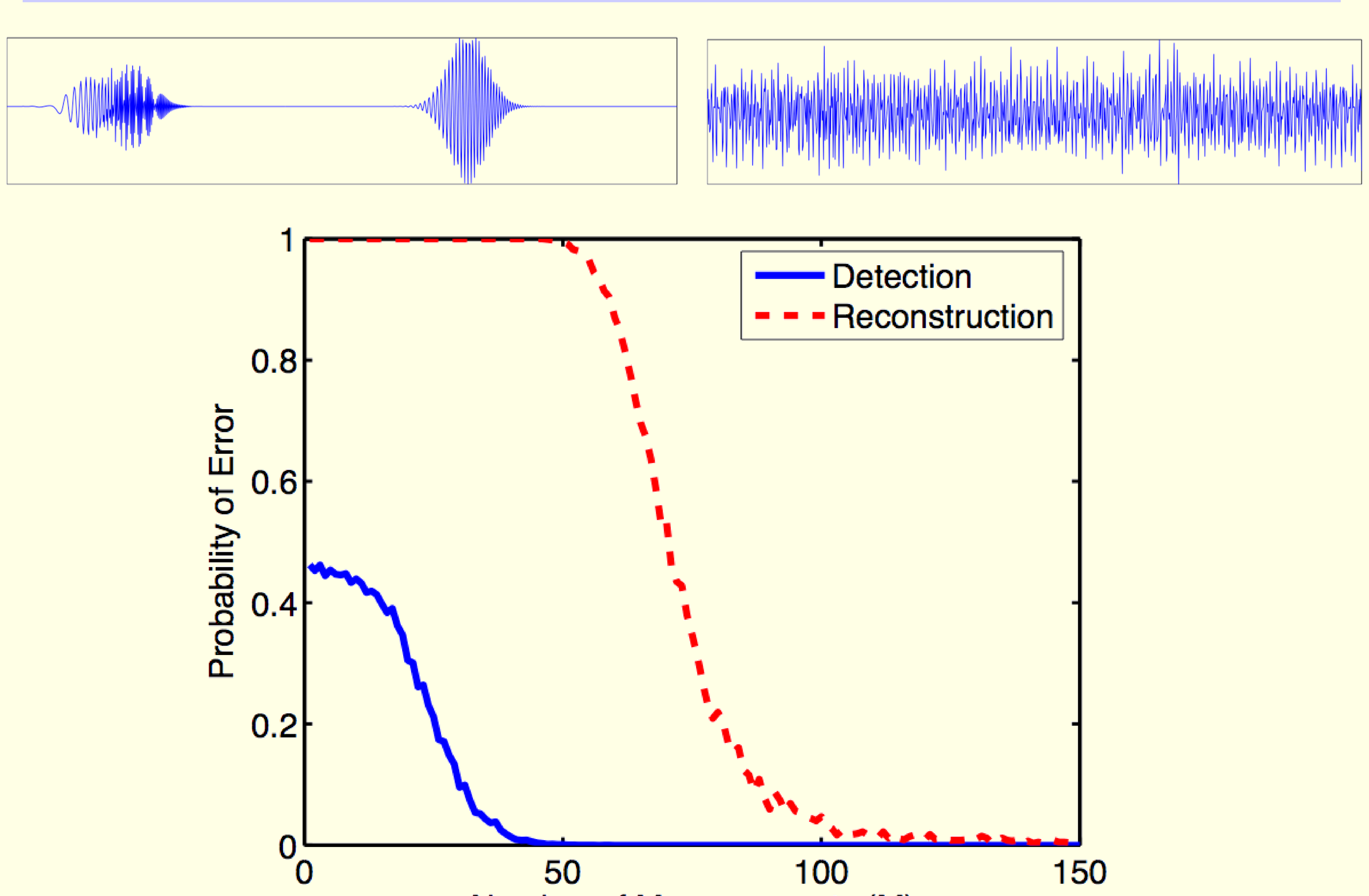
Universality of random projections

- one set of cK random projections contains enough information

From measurements $y = \Phi x$ use greedy algorithm on each class

- Orthogonal Matching Pursuit (OMP) stops when approximation is exact
- choose class for which OMP terminates first

Detection vs. Reconstruction



Detection requires

- *3x fewer measurements*
- *4x fewer iterations*

compared to MP reconstruction

Conclusions

Random projections as universal measurement scheme

- encode various types of signal statistics
- sparsity basis known only at decoder

Different algorithms can recover different levels of information from random measurements

- lower computational complexity
- fewer measurements needed

Incoherent Detection and Estimation Algorithm (IDEA)

- partial greedy pursuit
- detection without reconstruction
- extensible to other inference tasks

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