The Compressive Multiplexer

Compressive Sensing of Multichannel Signals

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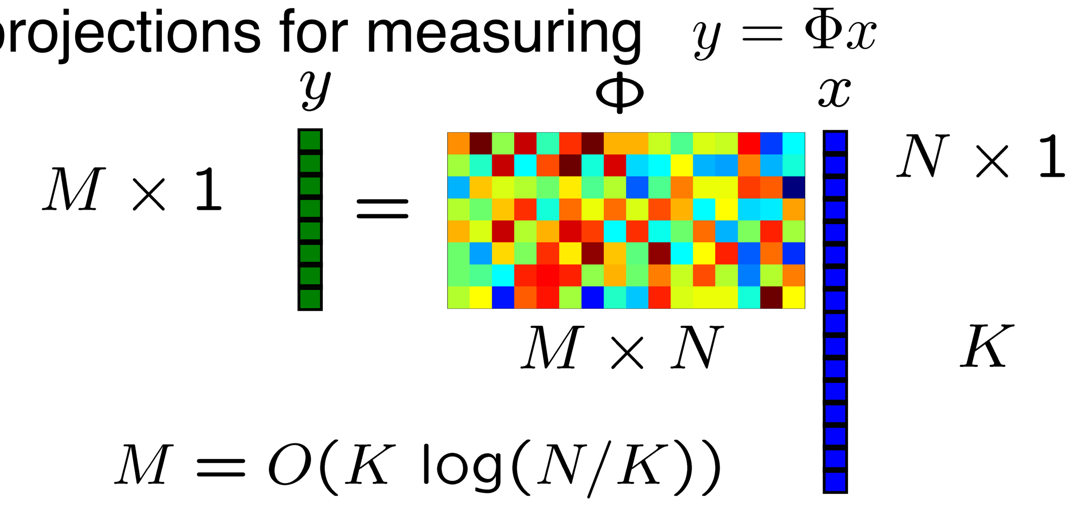
Compressive Sensing (CS)

The CS Framework

Random projections for measuring $y = \Phi x$



$M = O(K \log(N/K))$

Reconstruct data x from measurements y using ***convex optimization***

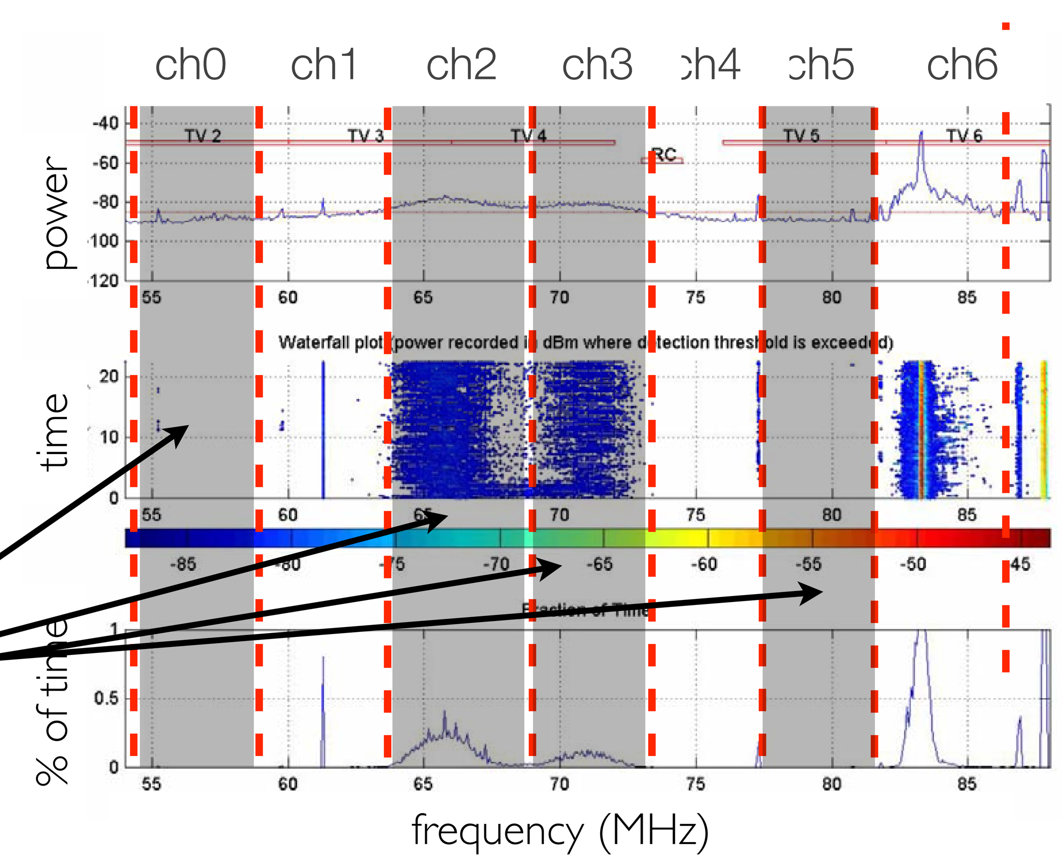
Wideband Multichannel Signals

Goal Acquire signals from multiple (potentially discontiguous) channels
- channels may be known *a priori*
- channels must be sparse when concatenated

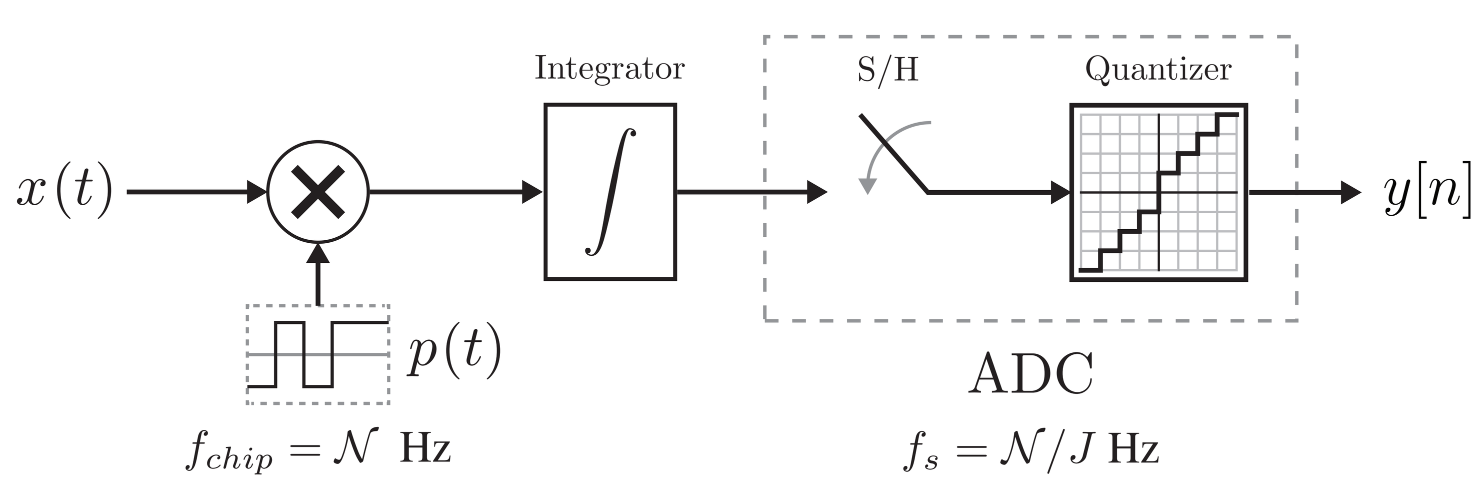
Example
wireless spectrum
most of spectrum not occupied

some channels not occupied

only interested in these channels

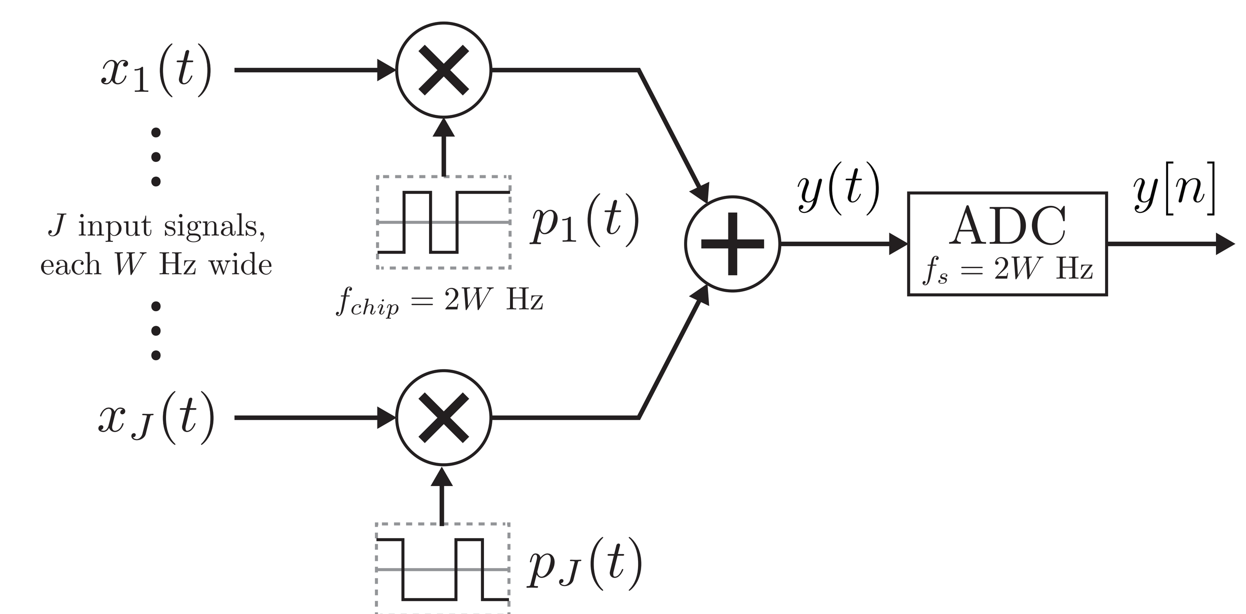


Random Demodulator (RD)

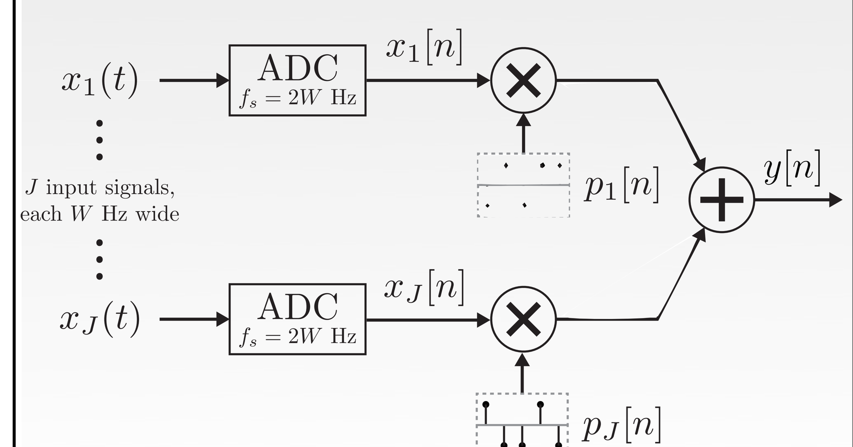


The Compressive Multiplexer

System Model



Equivalent System



CS Model

$$\Phi = \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\Phi_1}\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\Phi_2}\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\Phi_3}$$

$$A = [\Phi_1 \mathcal{F}, \Phi_2 \mathcal{F}, \cdots, \Phi_J \mathcal{F}]$$

recover α from $\mathbf{y} = A\alpha$

DFT matrix

Theory

Theorem Fix $\delta \in (0,1)$ There exists C_0 such that when

$$W \geq C_1 K \log^4(JW)$$

then A satisfies the RIP with probability $1 - C_0^2/\delta C_1^2$ [Neelamani, Romberg]

Algorithm Pairings

Trivial Reconstruction

$$\begin{aligned} \widehat{x}_i &= \Phi_i y \\ &= \Phi_i \left(\textstyle\sum_j \Phi_j x_j\right) \\ &= \underbrace{x_i}_{\text{channel i signal}} + \underbrace{\textstyle\sum_{i \neq j} \Phi_i \Phi_j x_j}_{\text{incoherent noise}} \end{aligned}$$

Block Coordinate Relaxation (BCR) [Sardy, Bruce, Tseng]

i) choose new channel index
$j \in \{1, \ldots, J\}$

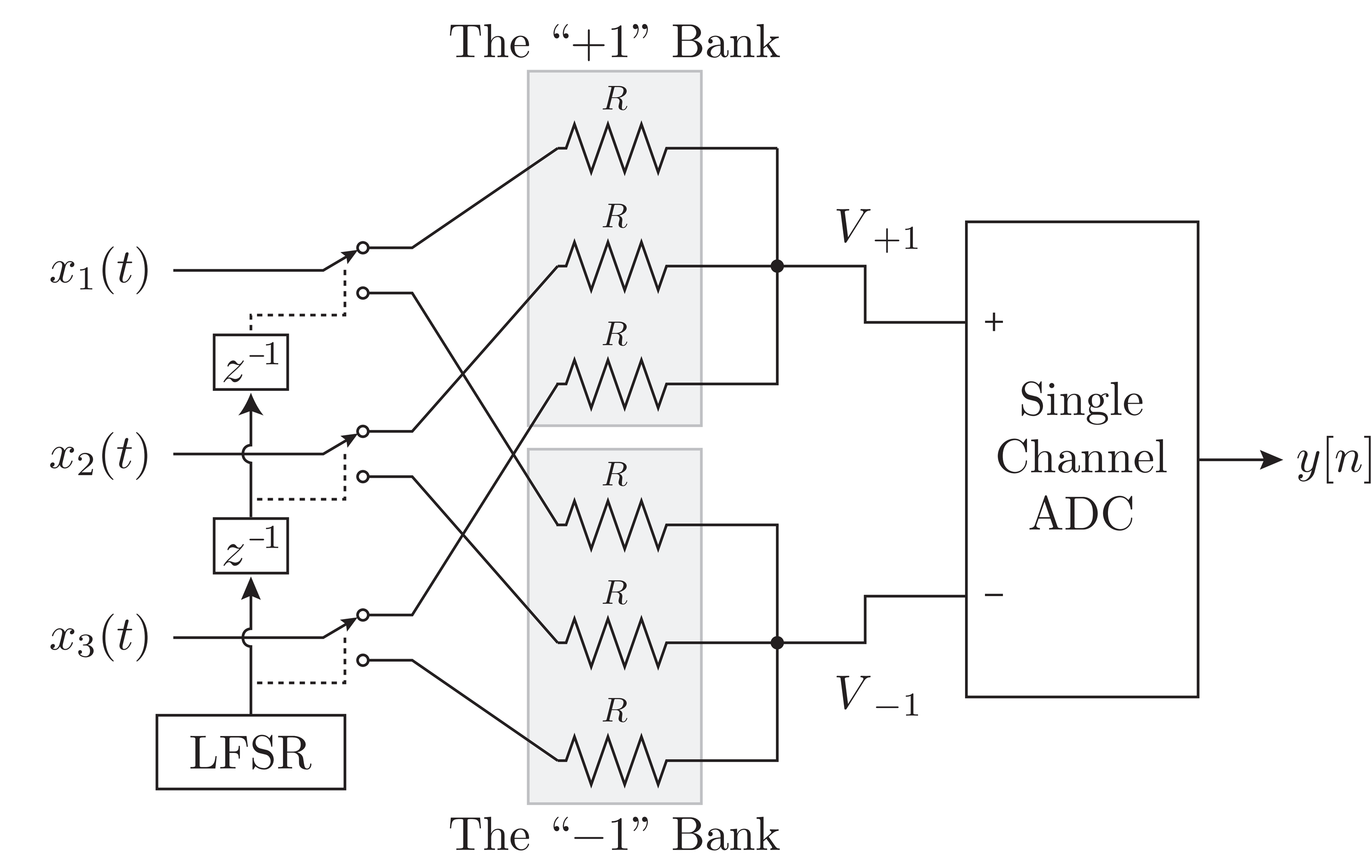
ii) subtract contribution from current estimate (except channel j)
$\mathbf{r} = \mathbf{y} - A_{\setminus j}\alpha_{\setminus j}$

iii) update channel coefficients
$\alpha_j = \mathcal{S}(A_j^T \mathbf{r}) \quad \mathcal{S}(\mathbf{z}) = \frac{\mathbf{z}}{|\mathbf{z}|}(|\mathbf{z}| - \lambda)_+$

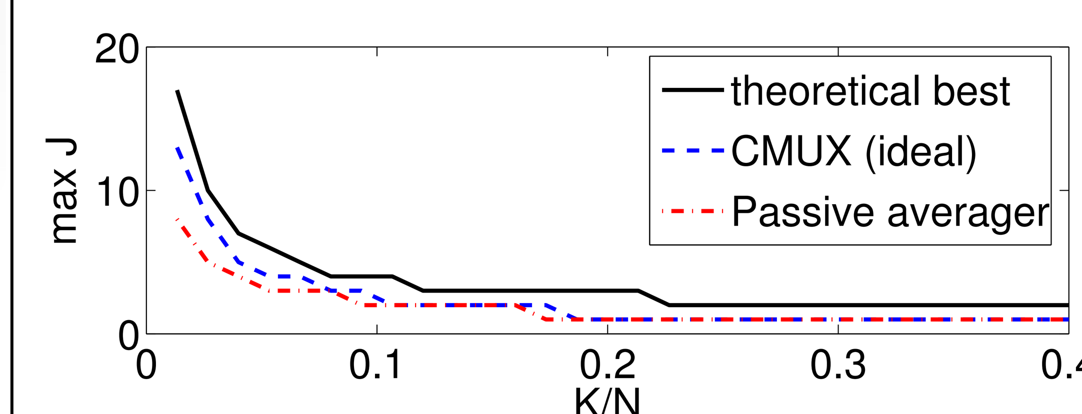
dsp.rice.edu/cs

Practice

The Passive Averager



Numerical Simulations



N = JW = 5000

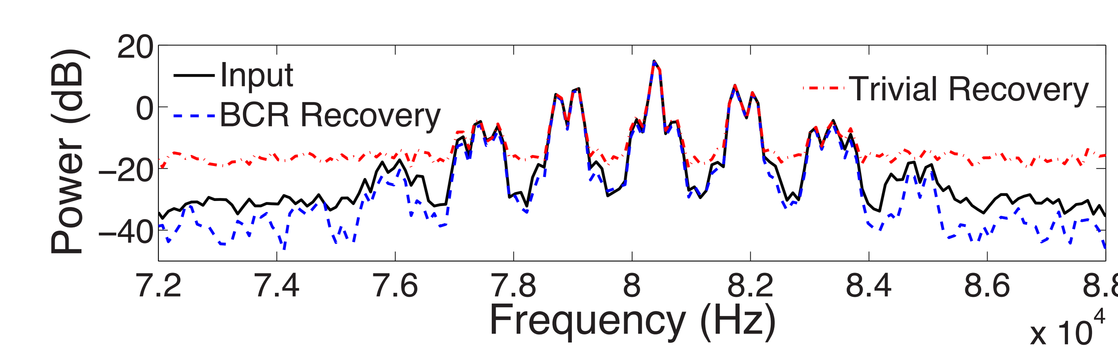
Search for best J, given K,N

90% recovery from 1000 trials

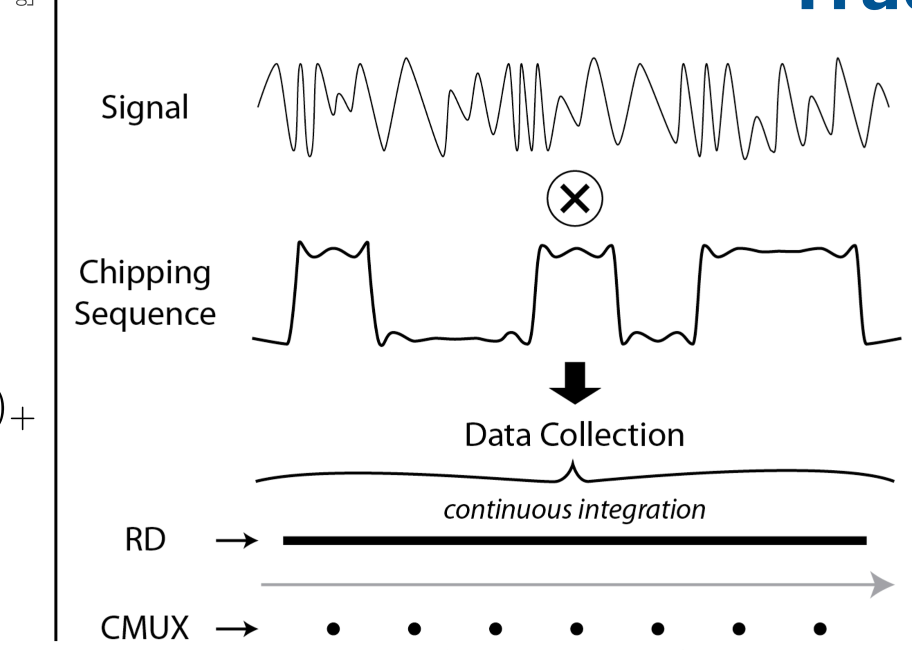
2 FM voice signals approximately 12 *KHz* wide (30dB SNR)

each signal in a different 400 *KHz* channel

5 channels total



Tradeoffs



CMUX does not integrate variations in chipping sequence waveforms

CMUX is easier to calibrate than the RD (no need to model filter)

CMUX subsampling rate is limited by number of channels