TEXAS HOLD 'EM ALGORITHMS FOR DISTRIBUTED COMPRESSIVE SENSING

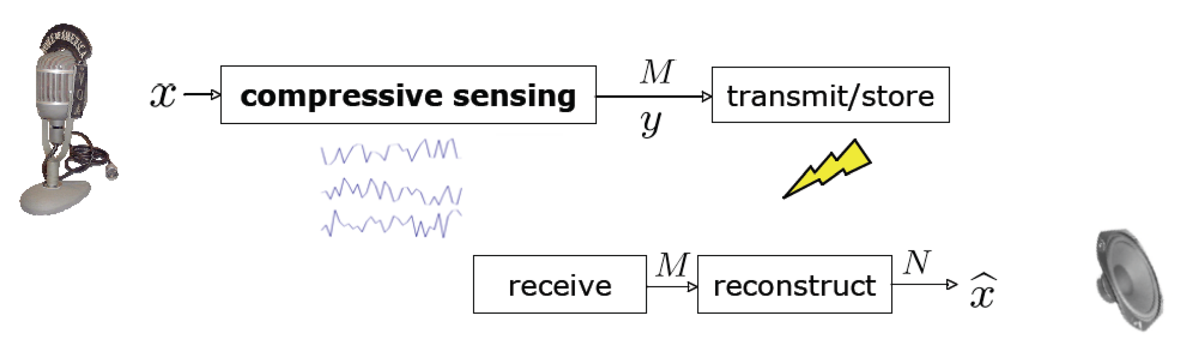
Stephen Schnelle, Jason Laska, Chinmay Hegde, Marco Duarte, Mark Davenport, Richard Baraniuk

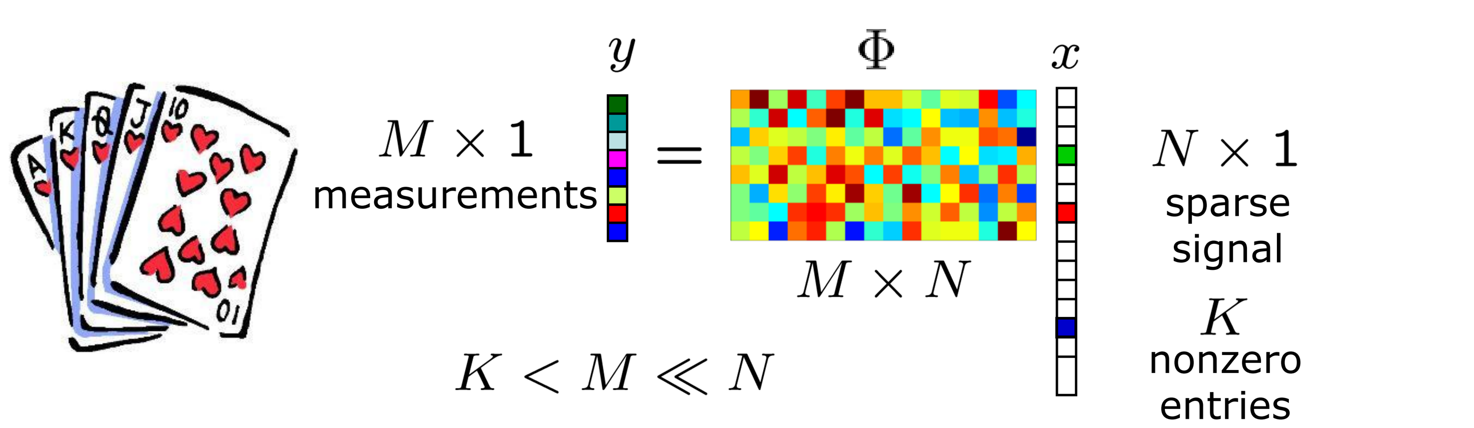
Rice University, Princeton University

CS and DCS

- CS samples sparse signals efficiently
- Joint sparsity models reduce total number of measurements correlated signals

Compressive Sensing





Restricted Isometry Property (RIP)

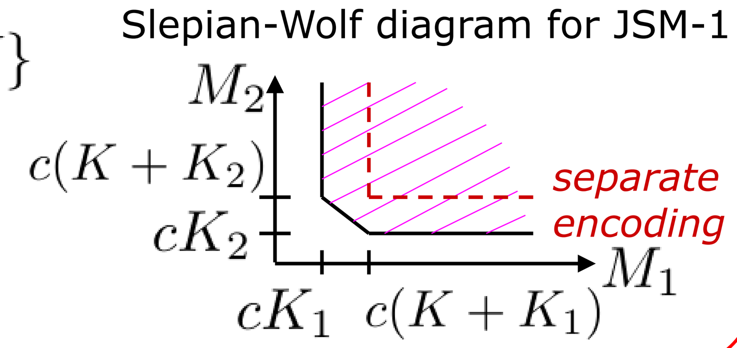
$$a\|\alpha\|_2^2 \leq \|\Phi\Psi\alpha\|_2^2 \leq b\|\alpha\|_2^2 \quad \forall\alpha \quad \text{s.t. } \|\alpha\|_0 \leq K$$

Joint Sparsity Models

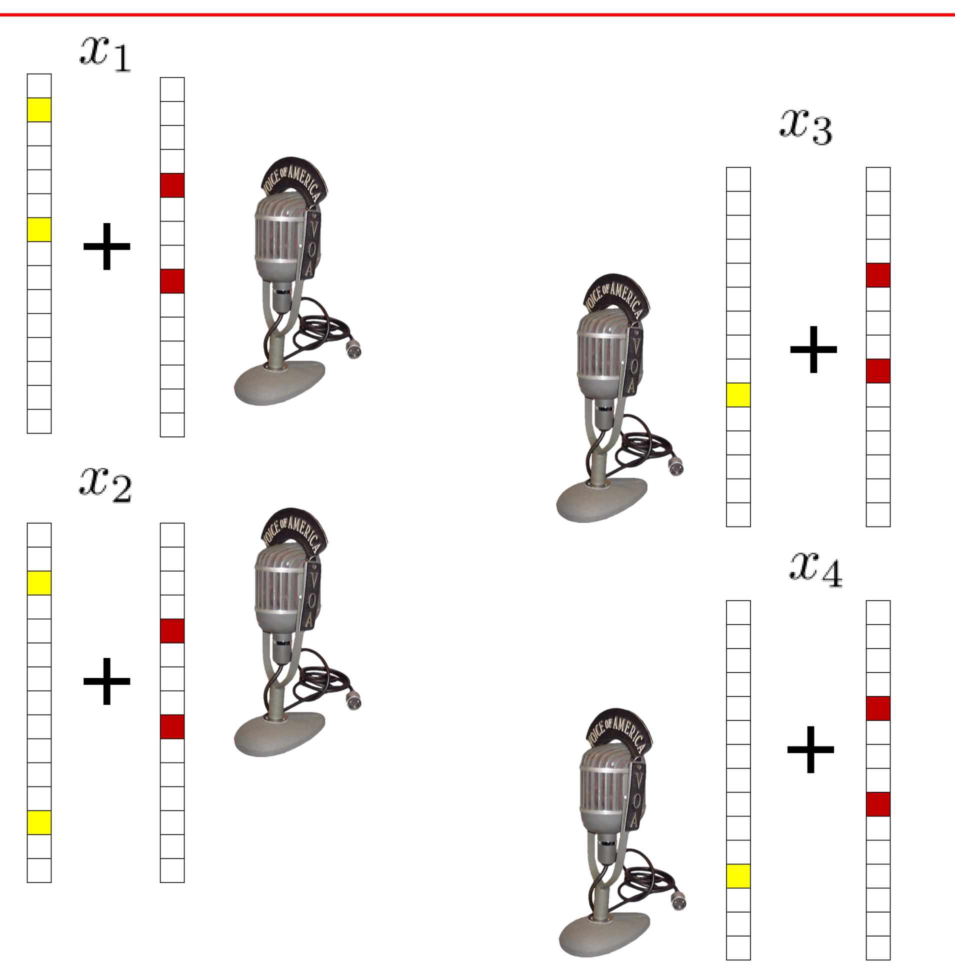
JSM-1: Sparse common component + innovations

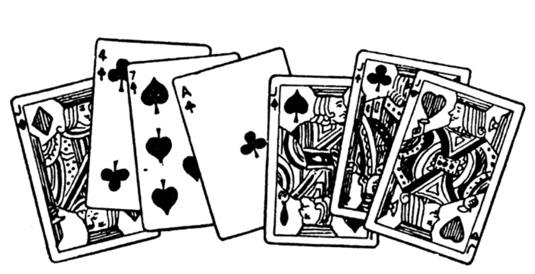
$$x_j = z + z_j, \quad j \in \{1, 2, \ldots, J\}$$
$$z = \Psi\alpha_z, \quad \|\alpha_z\|_0 = K$$
$$z_j = \Psi\alpha_j, \quad \|\alpha_z\|_0 = K$$

Slepian-Wolf diagram for JSM-1



e.g. temperature sensors





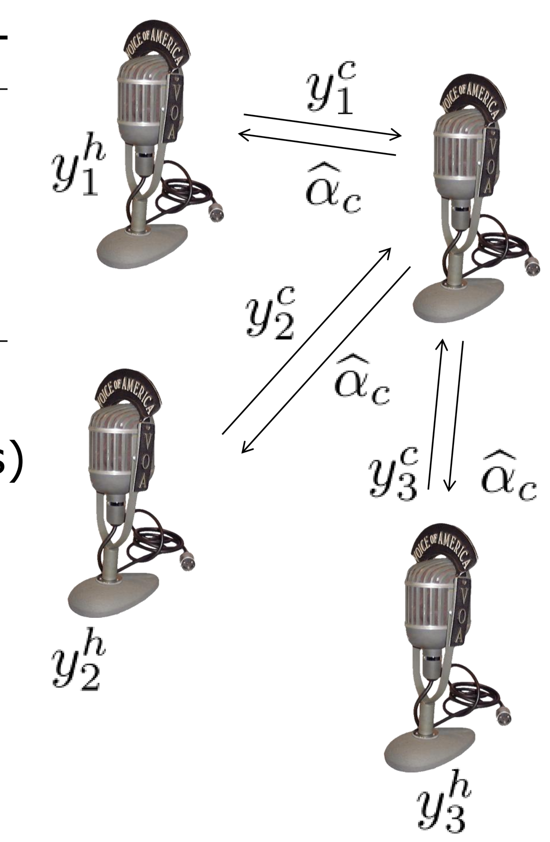
Texas Hold 'Em Algorithms

- Limits the number of measurements that must be exchanged among sensors
- Processing is handled locally

Main Algorithm

Algorithm 1 Texas Hold 'Em – Averaged Community

input: $\Psi, \bar{\Phi}, \bar{y}, K_c, \Phi_j, y_j, K_j$
$\widehat{\alpha}_c \leftarrow \text{RECOVER}(\bar{\Phi}\Psi, \bar{y}, K_c)$
$\widetilde{y}_j \leftarrow y_j - \Phi_j\Psi\widehat{\alpha}_c$
$\widehat{\alpha}_j \leftarrow \text{RECOVER}(\Phi_j\Psi, \widetilde{y}_j, K_j) + \widehat{\alpha}_c$
output: $\widehat{x}_j = \Psi\widehat{\alpha}_j$



- Decompose y into $[y_j^c \;\; y_j^h]$ (community and hold measurements)
- Reduce costs by only passing community measurements.
- Pass estimate of sparse common support back to calculate sparse innovations at each sensor

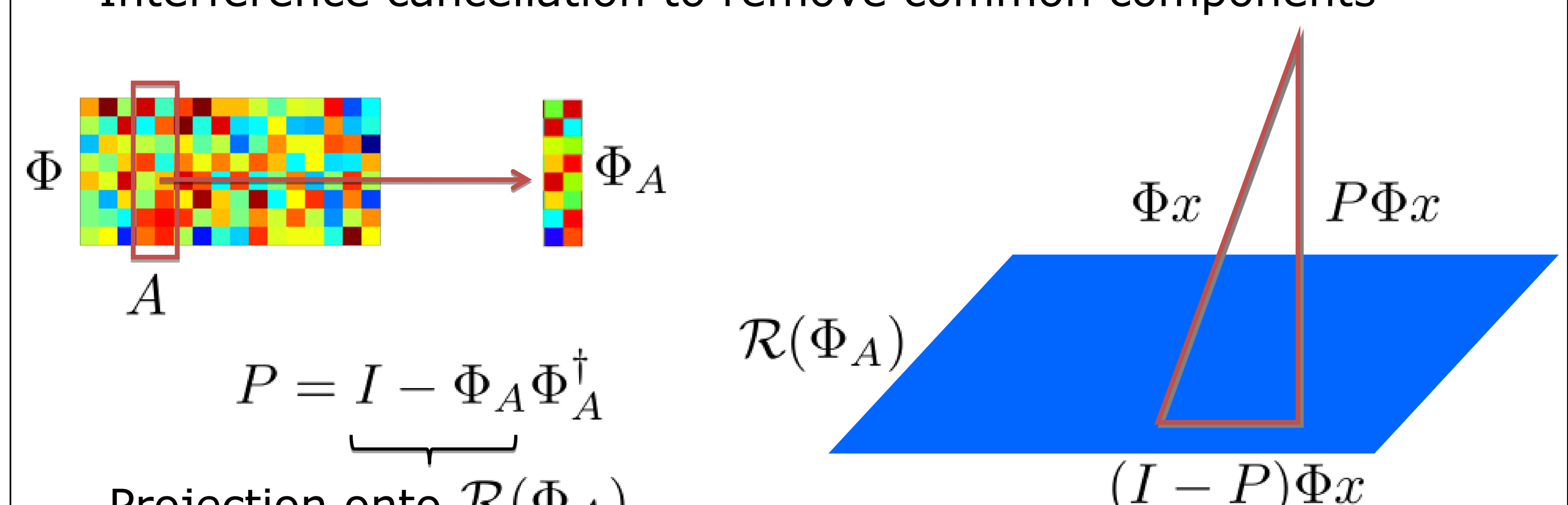
$$M = M_c + M_h$$

Average community measurements $\bar{y} = \sum_{j=1}^{J} \frac{y_j^c}{J}$

Common Community Measurement Matrix $\Phi_c = \bar{\Phi}$

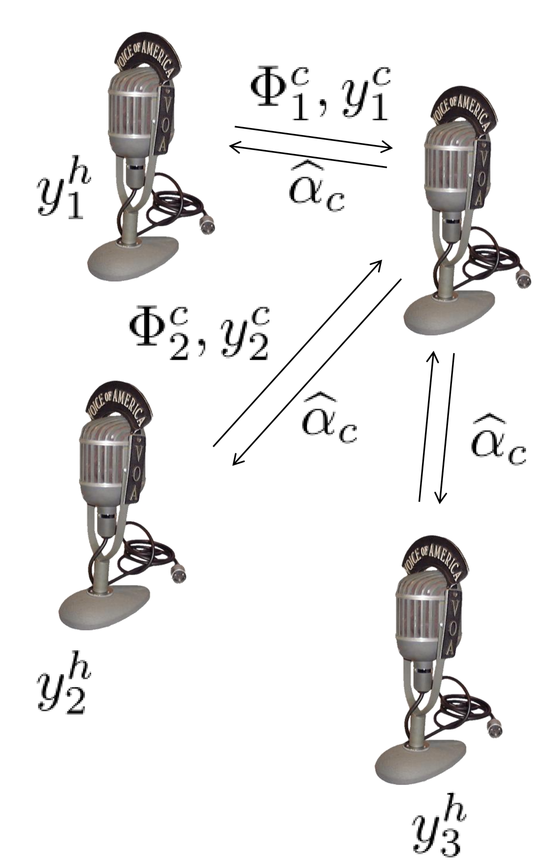
Variations

Interference cancellation to remove common components



$$\underbrace{P = I - \Phi_A\Phi_A^\dagger}_{\text{Projection onto } \mathcal{R}(\Phi_A)}$$

$$\bar{y} = \left[(y_1^c)^T, \quad (y_2^c)^T, \quad \cdots \quad (y_J^c)^T\right]^T$$
$$\bar{\Phi} = \left[(\Phi_1^c)^T, \quad (\Phi_2^c)^T, \quad \cdots \quad (\Phi_J^c)^T\right]^T$$



Algorithm 2 Texas Hold 'Em — Combined Community with Interference Cancellation

input: $\Psi, \bar{\Phi}, \bar{y}, K_c, \Phi_j, y_j, K_j$
$\widehat{\alpha}_c \leftarrow \text{RECOVER}(\bar{\Phi}\Psi, \bar{y}, K_c)$
$T_c \leftarrow \text{supp}(\widehat{\alpha}_c)$
$P_{T_c} \leftarrow I - \Phi_{jT_c}(\Phi_{jT_c})^\dagger$
$\widehat{\alpha}_j \leftarrow \text{RECOVER}(P_{T_c}\Phi_j\Psi, P_{T_c}\widetilde{y}_j, K_j) + \widehat{\alpha}_c$
output: $\widehat{x}_j = \Psi\widehat{\alpha}_j$

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Results

- ℓ_2 error directly proportional to ℓ_2 norm of sensor measurements
- ℓ_2 error inversely proportional to square root of number of sensors

Theory

Theorem: If

- $K = \max(2K_c, 2K_j)$
- Φ is an $M \times N$ random matrix fixed for all sensors with $M_c = O(K\log(N/K))$
- $\|z_c\|_0 = K_c, \; \|z_j\|_0 = K_j$
- $\|z_j\|_2 = \kappa$ for each j
- z_j are pairwise orthogonal

then w.h.p.,

$$\|\widehat{x}_j - x_j\|_2 \leq \frac{C'\kappa}{\sqrt{J}}$$

C' depends only on M, N and the choice of recovery method

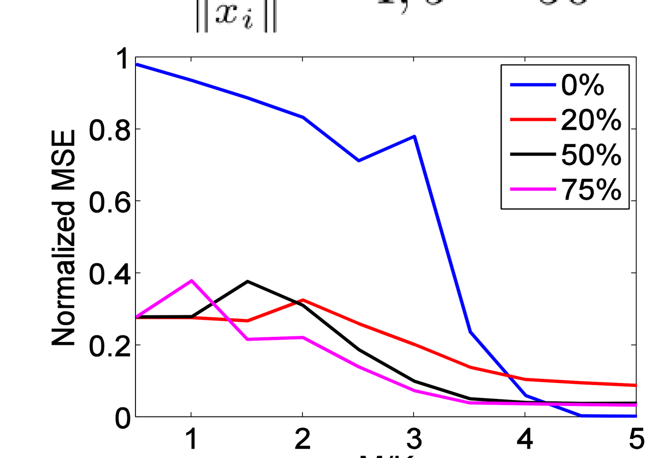
Proof:

$$\|\widehat{x}_j - x_j\|_2 \leq \|\widehat{z}_j - z_j\|_2 + \|\widehat{z}_c - z_c\|_2$$

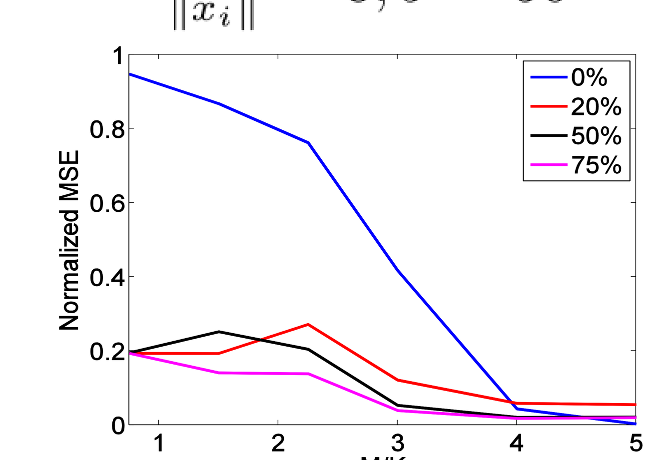
$$\|\widehat{z}_j - z_j\|_2 \leq \frac{C^2(1+\delta)\kappa}{\sqrt{J}} \qquad \|\widehat{z}_c - z_c\|_2 \leq \frac{C\sqrt{1+\delta}\kappa}{\sqrt{J}}$$

Simulations

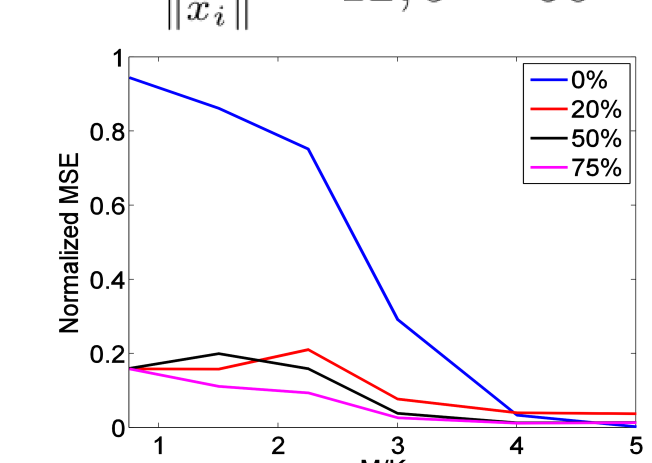
$\frac{\|x_c\|}{\|x_i\|} = 4, J = 50$



$\frac{\|x_c\|}{\|x_i\|} = 8, J = 50$



$\frac{\|x_c\|}{\|x_i\|} = 12, J = 50$



$\frac{\|x_c\|}{\|x_i\|} = 4, \frac{M_c}{M} = 50\%$