Compressive Sensing
Part I: Introduction

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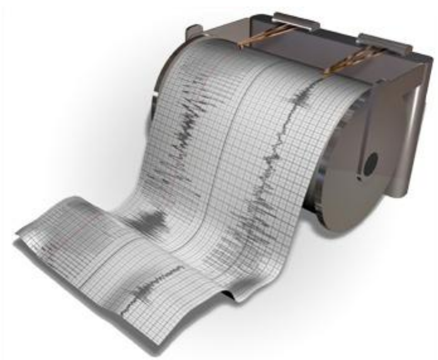
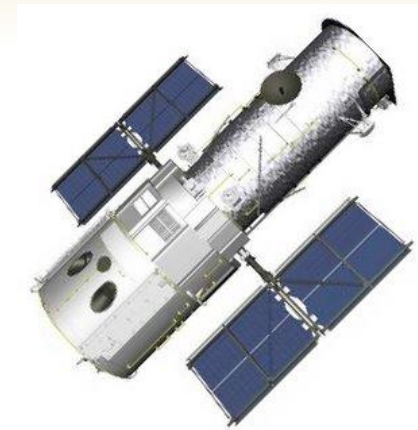
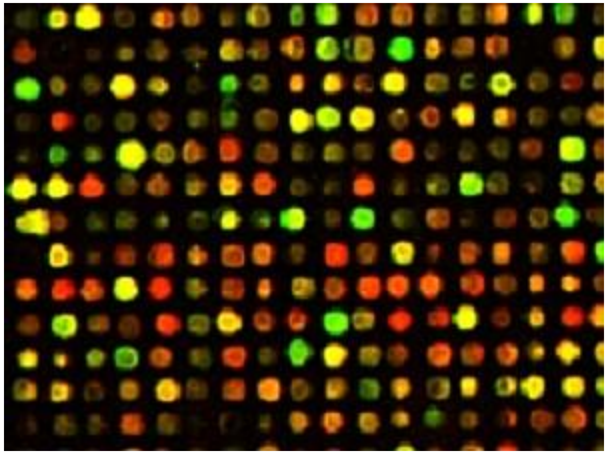
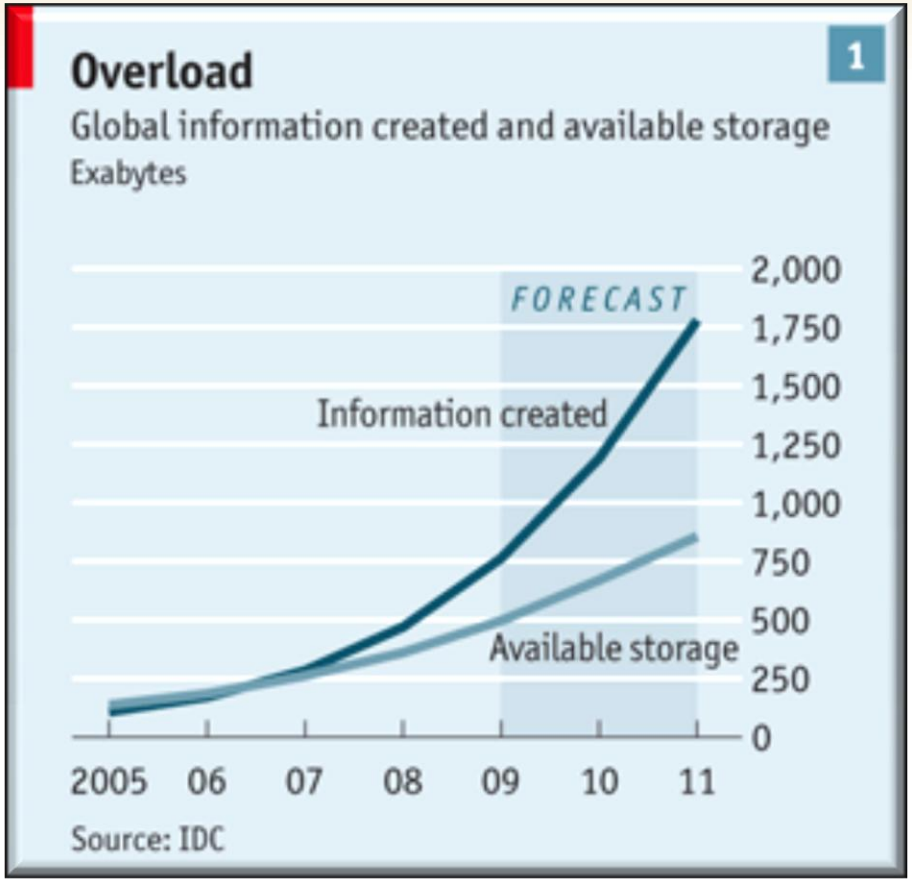
Sensor Explosion







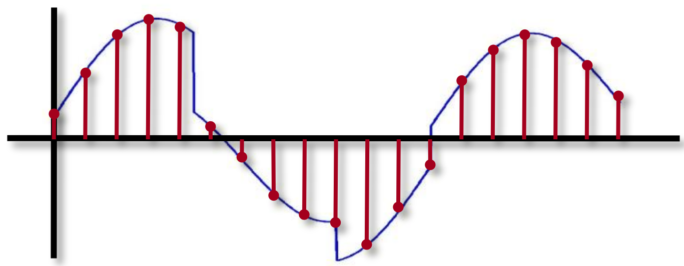
Data Deluge



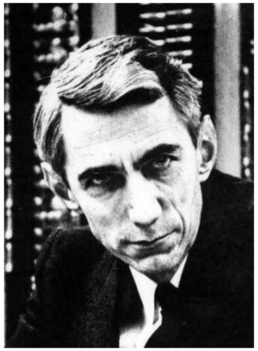
As of this year, ½ of digital universe has no home

[The Economist - 2010]

Digital Revolution

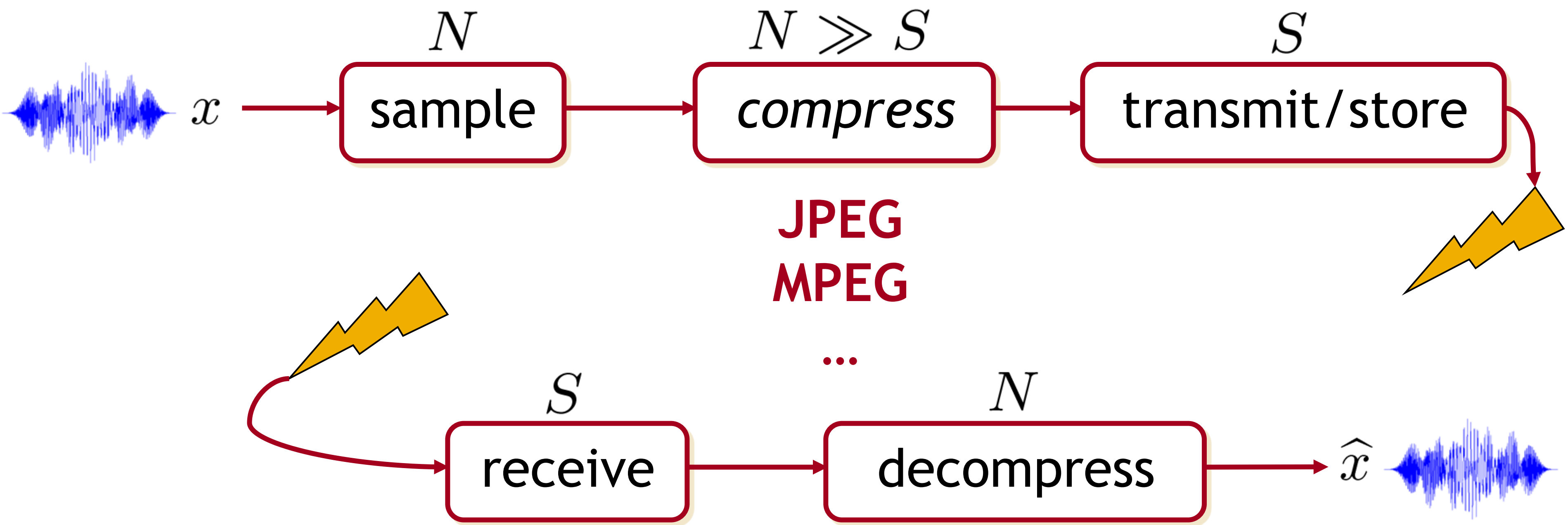


> "If we sample a signal at twice its highest frequency, then we can recover it exactly."
>
> Whittaker-Nyquist-Kotelnikov-Shannon



Sample-Then-Compress Paradigm

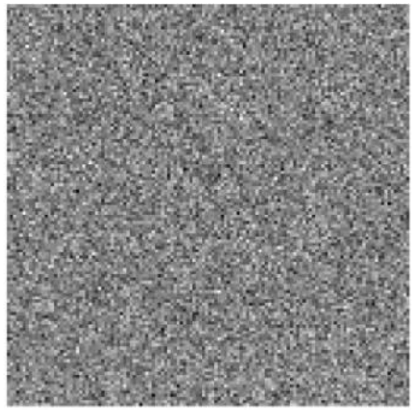
- Standard paradigm for digital data acquisition
  - ***sample*** data (ADC, digital camera, ...)
  - ***compress*** data (signal-dependent, nonlinear)



- Sample-and-compress paradigm is ***wasteful***
  - samples cost $$$ and/or time

Dimensionality Reduction

Data is rarely intrinsically high-dimensional



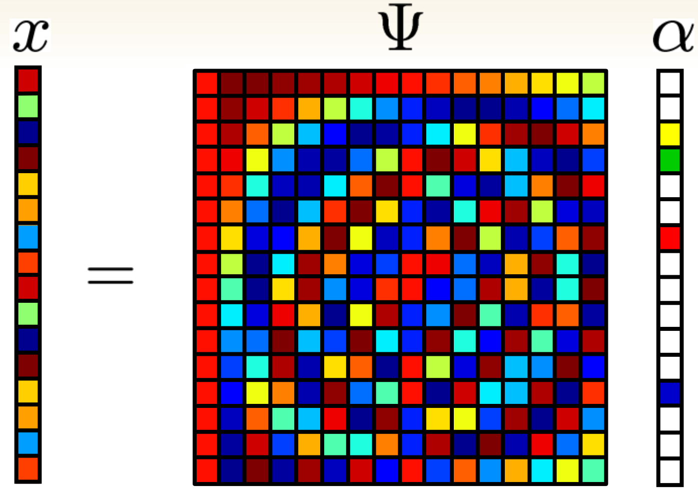
Signals often obey ***low-dimensional models***

- sparsity
- manifolds
- low-rank matrices

The "intrinsic dimension" S can be much less than the "ambient dimension" N

Sparsity

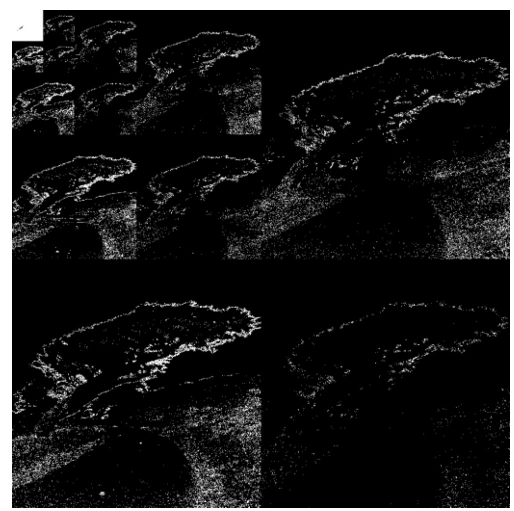
$$x = \sum_{j=1}^{N} \alpha_j \psi_j$$
$$= \Psi \alpha$$



S nonzero entries

$\|\alpha\|_0 = S$

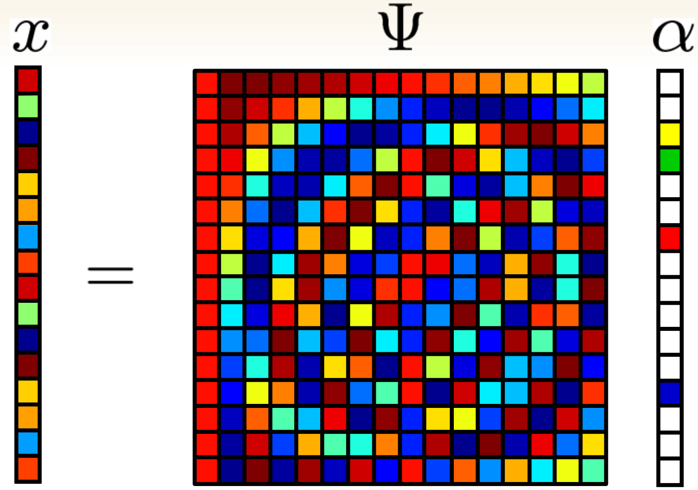
N pixels



$S \ll N$ large wavelet coefficients

Sparsity

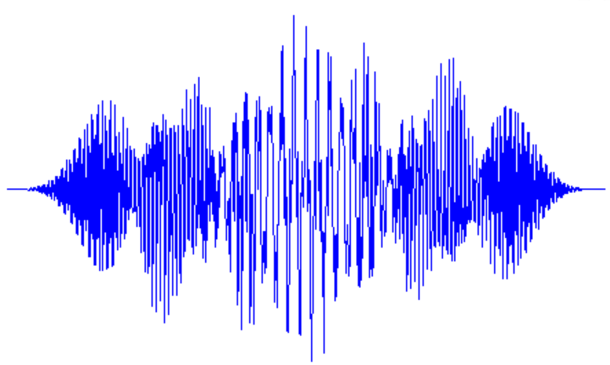
$$x = \sum_{j=1}^{N} \alpha_j \psi_j$$
$$= \Psi\alpha$$

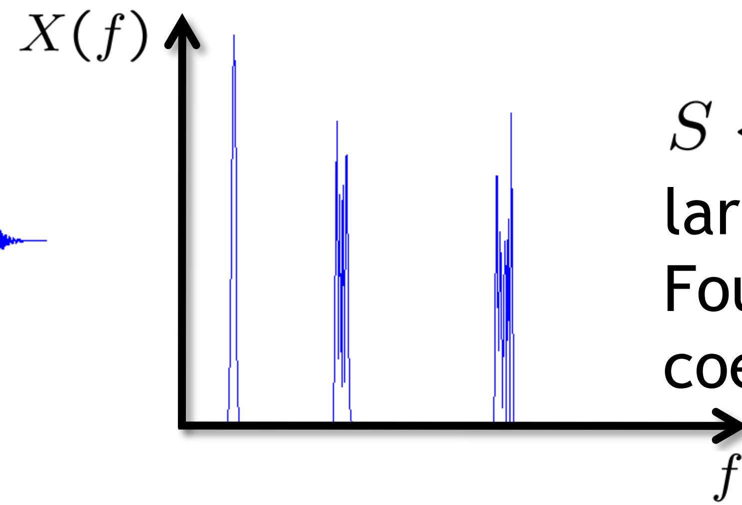


S nonzero entries

$\|\alpha\|_0 = S$

N samples

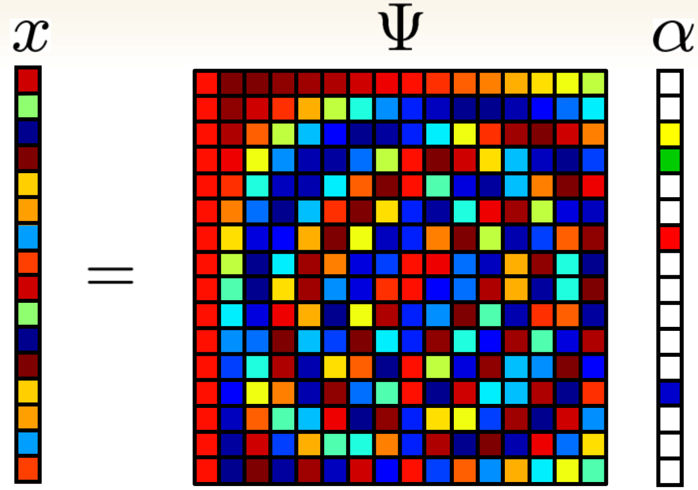




$S \ll N$ large Fourier coefficients

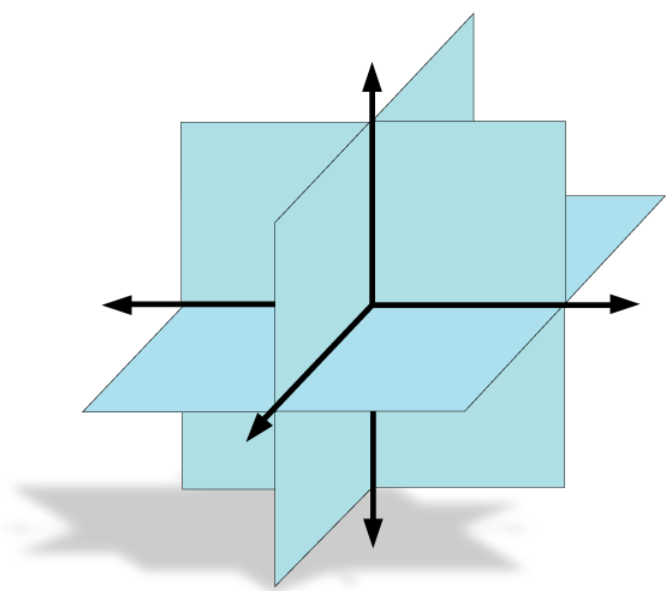
Sparsity

$$x = \sum_{j=1}^{N} \alpha_j \psi_j$$
$$= \Psi \alpha$$



S nonzero entries

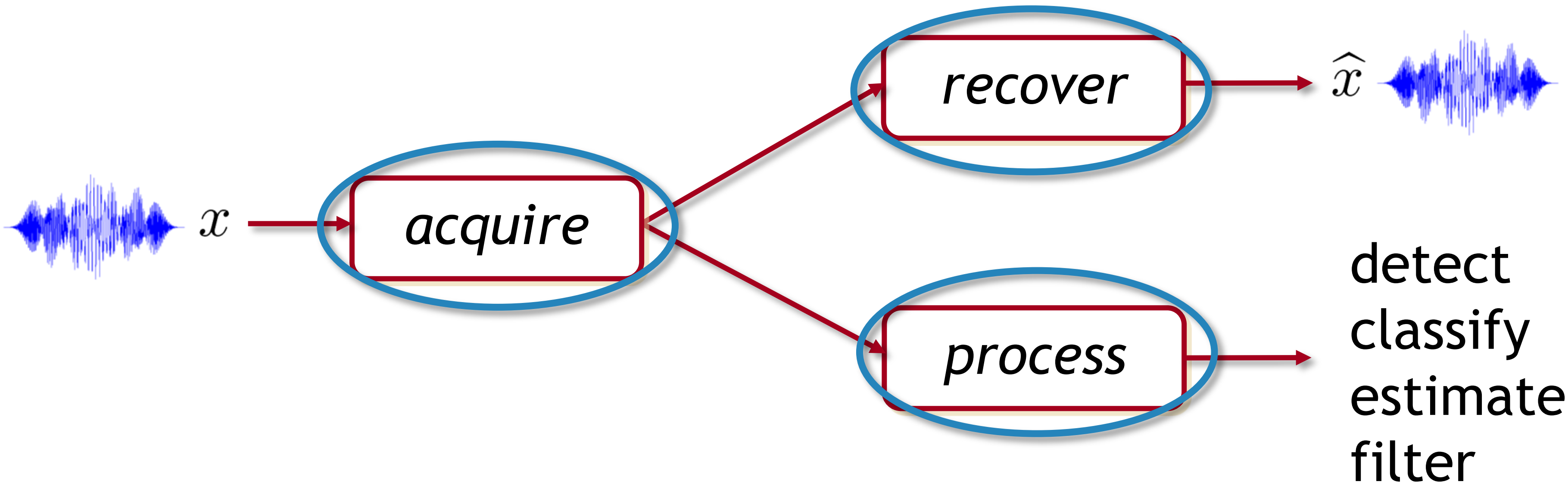
$$\|\alpha\|_0 = S$$



Exploiting Sparsity

How can we exploit sparsity in the design of signal processing algorithms?

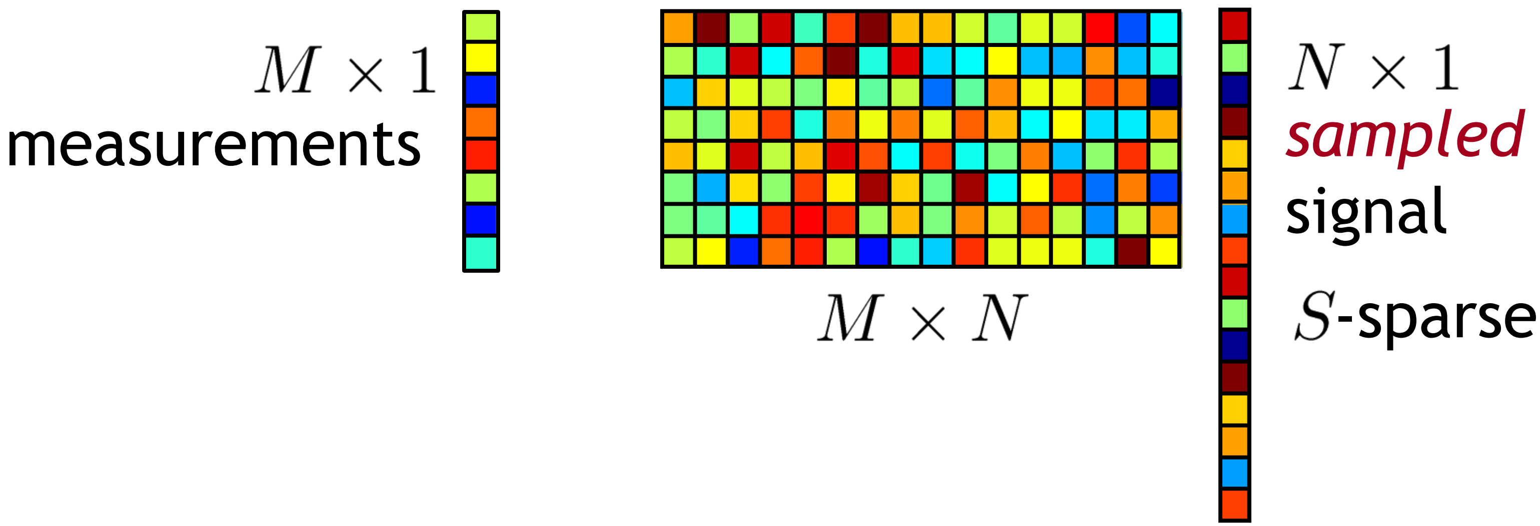
We would like to operate at the *intrinsic dimension* at all stages of the DSP pipeline



Compressive Sensing

Replace samples with general *linear measurements*

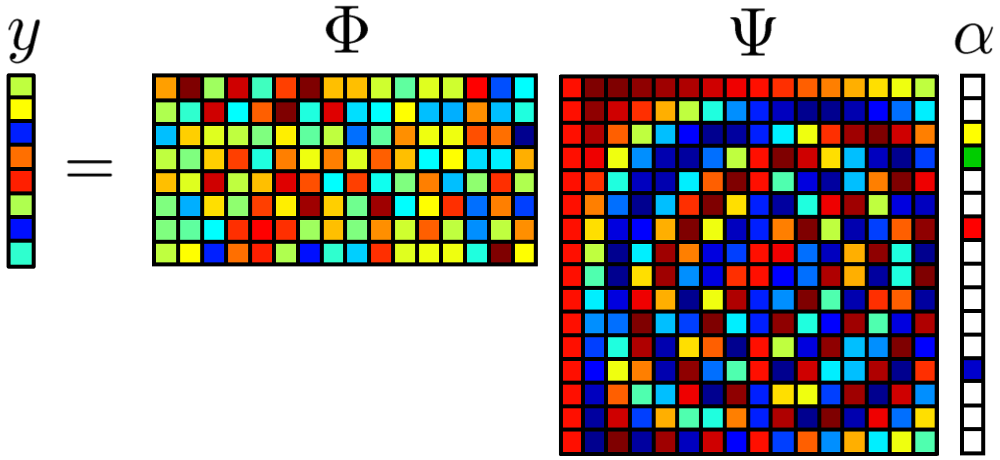
$$y = \Phi x$$



[Donoho; Candès, Romberg, and Tao - 2004]

Core Theoretical Challenges

- How should we design the matrix Φ so that M is as small as possible?



- How can we recover $x = \Psi\alpha$ from the measurements y ?

Outline

- Sensing matrices and real-world compressive sensors
  - (structured) randomness
  - tomography, cameras, ADCs, …
- Compressive sensing in practice
  - noise, interference, quantization, and dynamic range
  - real-world signal models
- Beyond sparsity
  - parametric models, manifolds, low-rank matrices, …
- Beyond recovery
  - compressive *signal processing*