

A Wideband Compressive Radio Receiver

M.A. Davenport, S.R. Schnelle, J.P. Slavinsky, R.G. Baraniuk
 Department of Electrical and Computer Engineering
 Rice University
 Houston, TX 77005

M.B. Wakin
 Division of Engineering
 Colorado School of Mines
 Golden, CO 80401

P.T. Boufounos
 Mitsubishi Electric
 Research Laboratories
 Cambridge, MA 02139

Abstract—Compressive sensing (CS) is an alternative to Shannon/Nyquist sampling for the acquisition of sparse or compressible signals. Instead of taking periodic samples, CS measures inner products with M random vectors, where M is much smaller than the number of Nyquist-rate samples. The implications of CS are promising for many applications and enable the design of new kinds of analog-to-digital converters, imaging systems, and sensor networks. In this paper, we propose and study a wideband compressive radio receiver (WCRR) architecture that can efficiently acquire and track FM and other narrowband signals that live within a wide frequency bandwidth. The receiver operates below the Nyquist rate and has much lower complexity than either a traditional sampling system or CS recovery system. Our methods differ from most standard approaches to the problem of CS recovery in that we do not assume that the signals of interest are confined to a discrete set of frequencies, and we do not rely on traditional recovery methods such as ℓ_1 -minimization. Instead, we develop a simple detection system that identifies the support of the narrowband FM signals and then applies compressive filtering techniques based on discrete prolate spheroidal sequences to cancel interference and isolate the signals. Lastly, a compressive phase-locked loop (PLL) directly recovers the FM message signals.

I. INTRODUCTION

Wideband radio receivers collect large segments of the radio frequency (RF) spectrum to detect, characterize, and extract information from signals of interest (SOIs). These devices are commonly used in both commercial and military systems to monitor signalling activity across a wide bandwidth. Ideally, such receiver systems should be able to monitor the entire bandwidth of interest and report on signals as soon as transmission is initiated.

Unfortunately, practical limitations often make such monitoring impossible. For example, collection of bandwidths on the order of 300–400 MHz demand analog-to-digital converters (ADCs) operating at rates close to 1 GHz. At these rates, state-of-the-art ADCs produce samples with relatively low dynamic range while also consuming significant power. The data rates and power requirements of such ADCs typically lead to increases in system size and weight. In place of a single ADC, alternative receiver implementations can be built with a parallel bank of low-rate ADCs. While increasing the achievable dynamic range, the extra hardware components also further increase the size, weight, and power consumption of the system. In addition to their hardware demands, both of these approaches also produce a deluge of data that can overwhelm downstream processing and information extraction algorithms.

To overcome these challenges, system designers typically build wideband receivers that scan through the spectrum one small segment at a time. Such receivers have smaller effective instantaneous bandwidth (EIBW), namely the usable bandwidth monitored at any time. This approach reduces the burden on the receivers' hardware and processing algorithms and makes "wideband" system behavior achievable in practice. The tradeoff, of course, is that signal energy cannot be captured if it is transmitted outside of the current tuning of the receiver's EIBW.

Compressive sensing (CS) is a relatively new framework for signal acquisition that offers a practical way to design wideband receivers that are able to monitor the entire bandwidth of interest without resorting to a scanning approach [1, 2]. Conventional Nyquist-based sampling assumes that the SOIs are bandlimited and dictates that in order to fully acquire the information in those signals the sampling rate needs to be at least twice the total collected bandwidth. CS assumes that rather than simply being bandlimited, the SOI is sparse in some basis, i.e., the signal has a small number of non-zero components when represented in that basis (see Fig. 5 (a) for a Fourier basis example). The underlying components can be recovered as long the sampling rate is proportional to the number of nonzero components [3]. In our setting, this means that a sample rate of at least $O(B \log(W/B))$ is sufficient, where B is the sum of the SOI bandwidths and W is the total collection bandwidth. While real-world signals are never truly sparse (they cannot be completely bandlimited and they are always embedded in a noisy environment), it is sufficient for the SOI to be *compressible*, meaning that it can be well-approximated by a sparse combination of bandlimited signals [4].

CS offers the potential for a massive data rate reduction, which can significantly reduce the requirements on a receiver system's hardware and processing algorithms. Thus, a single wideband compressive radio receiver (WCRR) can operate at a low data rate while still monitoring a very wide bandwidth as long as the total active signal bandwidth remains sufficiently small. However, current CS approaches typically rely on computationally expensive nonlinear algorithms to reconstruct the signal from the CS samples.

In this paper, we discuss the architecture and algorithmic approach for a WCRR that performs processing such as filtering and detection entirely in the compressive domain without first reconstructing the signal, thereby avoiding the

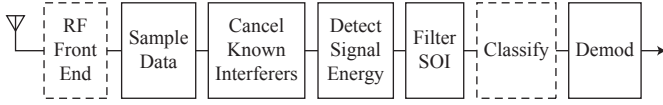


Fig. 1. Generic wideband receiver architecture. Dashed components are not addressed in this work.

brunt of the computational cost of reconstruction. We further show that in the case of FM signals we can directly demodulate and recover the baseband signal in the compressive domain. Experimental results show the outcome of applying these algorithms in simulated collection scenarios.

In the next section we describe the system architecture and the individual components of the WCRR. In Section III we present experimental results to demonstrate the performance of our system. We conclude with a discussion in Section IV.

II. WIDEBAND COMPRESSIVE RADIO RECEIVER COMPONENTS

A. Architecture

The generic architecture of a wideband receiver is shown in Fig. 1. The first component in the receiver consists of analog hardware to tune to and isolate the wide collection bandwidth. The analog output of this RF front end is then converted to digital samples. In a conventional receiver, the rate at which these samples are acquired must exceed the Nyquist rate of the analog signal, while the WCRR will sample at a much lower rate as described below. After optionally nulling out the effects of known interfering signals, the receiver detects the presence of signals in the monitored bandwidth. In the case of single-channel receivers, numerous techniques including spectral energy detection can be used. The receiver usually has some *a priori* information about the SOIs. For example, a directed receiver knows precisely where to look for signals and only needs to detect if the signal is present. Once a SOI has been detected, the receiver filters it, classifies its type, and applies the appropriate demodulation to extract the necessary information from the signal.

In the sections below, we outline and describe a WCRR that processes signals entirely in the compressive domain, without the need for the computationally expensive reconstruction algorithms normally used in CS systems. We demonstrate methods for compressive sampling, interference cancellation, signal detection, filtering, and demodulation. RF front end processing and compressive signal classification are not discussed. The former is a well-established technology, while the latter has been explored for more general scenarios in several previous publications such as [5, 6].

B. Compressive Sampler

Of the several practical hardware architectures for compressive sampling, the receiver proposed in this work uses the random demodulator architecture [7]. Other candidates include random filtering [8], random sampling [9], and random convolution [10, 11]. The canonical sub-Gaussian random matrix

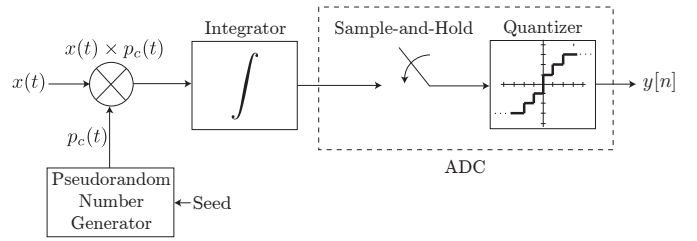


Fig. 2. Random demodulator block diagram.

approach [12] is highly useful as a gold standard methodology but is typically impossible to implement in practical receivers.

The four key components of a random demodulator, illustrated in Fig. 2, are (i) a pseudo-random ± 1 “chipping sequence” generator, (ii) an analog mixer (or modulator) to modulate the input signal by the chipping sequence, (iii) an integrate-and-dump circuit, and (iv) a low-rate ADC composed of a simple sample-and-hold circuit followed by a quantizer. While the back-end ADC operates at a low rate, the chipping sequence and the modulator must operate at or above the Nyquist rate of the system’s collection bandwidth W . With current technology it is easier to build a high-rate modulator in hardware than a high-rate ADC. In fact, many systems, most notably CDMA communication devices, already use components of this front end.

Mathematically, we can equivalently describe our sampling system as a linear operator Φ that maps Nyquist rate samples of $x(t)$ to a discrete sequence of measurements $y[n]$ and then quantizes them. In this work we assume that all processing is performed on a finite window of data, in which case the finite set of Nyquist rate samples are simply a vector in \mathbb{R}^N . This leads to a finite window of M measurements. Using this discrete formulation, the random demodulator is equivalent to multiplying the Nyquist rate samples $x[n]$ with a random sequence of ± 1 ’s and then summing every N/M sequential coefficients to obtain M measurements. In this case Φ can be represented as an $M \times N$ matrix where $N = LM$, with L representing the decimation factor, or the ratio of the Nyquist rate to the sampling rate of the ADC in the random demodulator. For example, in the case where $M = 3$ and $L = 3$, we can write Φ as the 3×9 matrix

$$\Phi = \begin{bmatrix} -1 & 1 & -1 & & & & & & \\ & & & -1 & -1 & 1 & & & \\ & & & & & & 1 & 1 & -1 \end{bmatrix}.$$

The fact that we can express Φ simply as a matrix applied to a window of Nyquist rate samples will prove important throughout this paper, as it will allow us to develop a number of simple detection, interference cancellation, filtering, and demodulation schemes.

C. Detection

We now turn to the problem of detecting approximately where the SOIs lie within the collection bandwidth. This will allow us to further filter and process the data to recover each

SOI. The signal energy detector operates on the vector y of M samples obtained from the compressive sampler. Let t denote the vector of time values corresponding to the sampling locations for the implicit Nyquist rate sampled version of x . We then consider a grid of possible frequency values $\{f_k\}_{k=1}^K$ and compute

$$S(k) = |\langle \Phi e^{j2\pi f_k t}, y \rangle|^2 = |y^T \Phi e^{j2\pi f_k t}|^2 \quad (1)$$

for $k = 1, \dots, K$, i.e., we simply correlate y against a series of vectors that are the outputs of the compressive sampler given pure sinusoid inputs. The result is a length- K vector S that provides an estimate of the power in the measurements at a sequence of frequencies of interest. This step is reminiscent of the first step of many greedy algorithms for CS recovery, for example see [13, 14]. Clearly, this sequence of frequencies should be sufficiently dense to be able to detect narrowband pulses. On the other hand, the spacing between frequencies should be relatively large in order to reduce the required number of computations. This allows for a tradeoff between the accuracy of the estimate S and the speed of its computation.

Once we have computed S , the detection problem can be solved via a variety of classical techniques. In our experiments we focus on a simple thresholding of S to identify the center frequencies of the detected signals.

D. Interference Cancellation and Filtering

Wideband radio receivers handle multiple narrowband signals within a large bandwidth; in order to perform further processing on a particular SOI, we must be able to filter out the remaining narrowband signals that are not of interest. *Interference* could also arise when some of the narrowband signals are extraneous, or possibly just not of interest at the time. For example, a television signal could represent high-power interference if it lies in the collection bandwidth. Or we may just wish to focus on a particular signal or frequency band for demodulation and ignore others, such as monitoring a particular communication band. In each scenario, we can decompose the signal x as $x = x_S + x_I$, where x_S is the desired signal and x_I represents any component not of interest at the moment.

The procedure for filtering in the compressive domain is different than standard bandpass filtering. Rather than designing an FIR or IIR filter for a given sampling frequency, a projection matrix is computed such that the spectral components of the interfering signal lie in its nullspace.

In order to do this, we must first obtain a basis for the subspace in which the interference lives. We will model interference as a length- N window of a bandlimited signal (with band limits f_1 and f_2). Strictly speaking, such signals do *not* live in a subspace of \mathbb{R}^N , but one can show that they live very close to a low-dimensional subspace spanned by the first J *discrete prolate spheroidal sequences* (DPSS's) [15]. The DPSS's are the finite-length vectors that are simultaneously most concentrated in time and in frequency on the desired baseband bandwidth. In general, we can generate N DPSS's, but typically $J \ll N$ is sufficient to capture most of the energy

in a bandlimited function. While there do exist rules of thumb for setting J , we will leave J as a parameter to be set by the user, with larger J allowing for better suppression of the undesired signal but also leading to slightly more distortion of the desired signal. We will let Ψ denote the $J \times N$ DPSS basis, which is generated by modulating baseband DPSS's by a cosine of frequency $(f_2 - f_1)/2$ [16]. If we have multiple interferers, then we can simply generate a basis for each and concatenate them into a matrix Ψ .

Once we have obtained the basis Ψ , interference cancellation can be performed as detailed in [17] by forming the matrix

$$P = I - \Phi \Psi (\Phi \Psi)^\dagger, \quad (2)$$

where A^\dagger denotes the pseudoinverse $A^\dagger = (A^T A)^{-1} A^T$. This will cancel the interference while preserving the SOI.

Note that the matrix P in (2) is an orthogonal projection. This is particularly useful, since after filtering we have $P y = P \Phi x$. One might expect that in order to perform the detection strategy described in (1), we would need to compute $P \Phi e^{j2\pi f_k t}$. However, since P is an orthogonal projection matrix we obtain

$$\begin{aligned} \langle P y, P w \rangle &= w^T P^T P y \\ &= w^T P^2 y \\ &= w^T P y \\ &= \langle P y, w \rangle. \end{aligned}$$

Therefore, we can use the same detection strategy as in Sec. II-C *after* we have filtered the measurements without any modifications. This will also be an extremely useful fact when attempting to demodulate the SOIs, as described in the following section.

E. Demodulation

For demodulation of FM modulated signals, we introduce a new family of phase-locked loops (PLLs) based on compressive sensing. As shown in Fig. 3, a conventional PLL computes the phase difference between an input signal and a PLL-produced oscillator signal. This phase difference is used in a feedback loop to update the phase of the oscillator. In this way, the PLL tracks the phase of the input signal.

The phase detector in a PLL can be interpreted as computing a weighted inner product between the Nyquist-rate input samples $x[n]$ and the estimated signal samples $u[n]$ generated by the oscillator. The restricted isometry property (RIP) of CS says that the standard inner product between $x[n]$ and $u[n]$ will be very close to that of the compressively sampled versions of $x[n]$ and $u[n]$ [6]. Inspired by this property, we construct a compressive sensing phase-locked loop (CS-PLL) by inserting two compressive samplers into the conventional PLL: one operating on the analog input signal $x(t)$ to produce the CS samples $y[m]$ and another operating on the discrete-time oscillator signal to produce $v[m]$ [18]. The index m indicates the lower compressive sample rate while n indicates sampling at the input signal's Nyquist rate. Connecting the

CS-PLL into the receiver architecture, the phase detector's $y[m]$ input is simply the output of the receiver's random demodulator component.

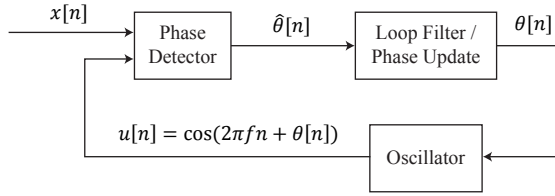


Fig. 3. Conventional PLL block diagram

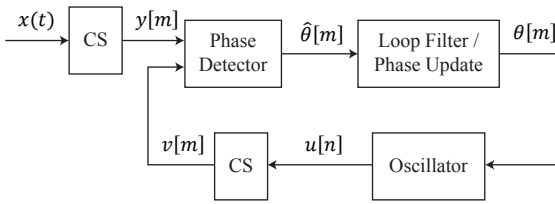


Fig. 4. CS-PLL block diagram

To conserve the inner product as indicated by the RIP, efforts need to be taken to ensure that the CS-PLL's two compressive sampling operations are as similar as possible. We achieve this by inputting the same pseudo-random sequence to the two multipliers, using the same impulse response in the two integrators, and by synchronizing the low-rate sampling in the two ADCs. Additionally, the sampling operation in the feedback loop must be aligned appropriately to account for any processing delays that occur between the random demodulator and the CS-PLL. In the setup shown in Fig. 4, the compressive samplers have both analog and digital inputs; some calibration may be required, but in general the behaviors of these two samplers will resemble one another.

As with the classical PLL, the CS-PLL computes the phase estimate inner product using a multiplier and a filter. Generally, the filter can be nonlinear and/or time-varying. In our receiver, we model the compressive sampler digitally and use a linear time-invariant filter. This filter mimics the characteristics of the loop filter in the high-rate traditional PLL, but with frequency scaling to adjust the filter bands for the new lower sample rate. The resulting phase estimate is given by

$$\hat{\theta}[m] = \sum_k y[k]v[k]h[m-k], \quad (3)$$

where $h[m]$, the linear filter impulse response, acts as the kernel of the weighted inner product.

PLLs are designed to obtain the frequency and/or phase of the input signal after a reasonable number of iterations and to retain this information as the signal progresses through time. For applications such as FM demodulation, the baseband message signal can be read directly from the error signal $\theta[m]$ of the PLL when the correct frequency is maintained.

When the PLL is outputting the desired signal it is said to be “locked.” Notably, the error signal $\theta[m]$ is produced at the lower compressive sample rate and not the Nyquist rate of the signal produced by the oscillator.

Out-of-band noise issues are easier to address in conventional PLLs vs. CS-PLLs. Systems typically have an estimate of the target signal's center frequency prior to applying a PLL (in our receiver this value is output from the detection stage). Since PLLs work on relatively narrowband signals, a conventional PLL can use this frequency estimate to apply a traditional bandpass filter that eliminates out-of-band noise ahead of PLL processing. For a CS-PLL this problem is not so simple. If we know the center frequency prior to compressive sampling, then we can apply an analog bandpass filter similar to the conventional system's digital bandpass filter. However, this solution is less desirable as (i) tunable analog filters are not as easy to incorporate and (ii) by filtering at the front-end we lose the ability to monitor the entire bandwidth of interest. Hence, aside from the ability of the previously mentioned interference cancellation and filtering algorithms, out-of-band noise will unavoidably contribute to the error in recovered signal. This is similar to the “noise-folding” effect that has been observed in traditional CS recovery [3].

III. EXPERIMENTAL EVALUATION

The principal inspiration for our experiments is to show that the algorithms discussed in this paper can be effectively combined into a system that performs the job of a traditional wideband receiver but at a greatly reduced data rate. The assumptions we make in our setup are motivated by the desire to simply demonstrate the feasibility of the system. The reader will likely see many improvements that can (and should) be made to increase the utility and robustness of the receiver and its algorithms in real-world operational contexts; some of these are discussed at the end of this work. In many cases, these improvements do not change the fundamental algorithmic work under exhibit here.

A. Scenario

In the simulation results that follow, the WCRR monitors an EIBW of 300 MHz in which 5 FM-modulated voice signals are transmitting at carrier frequencies unknown to the receiver. There is also a 140 kHz wide interferer at 200 MHz that is at least 35 dB stronger than any other signal. Each signal occupies roughly 12 kHz of bandwidth. The 300 MHz bandwidth is modeled as being acquired from an aerial platform operating at a distance of 8 miles from the target signals with line of sight to each signal. The WCRR system compressively samples at a rate of 30 MHz (20 times undersampled).

Our experiments assume an input of a tuned and down-converted RF signal. Additionally, we assume the signal is collected in time-limited blocks, that the captured signals are known to be FM modulated, that there are a known number of signals in the collection bandwidth, and that the signals are separated by at least 30 kHz.

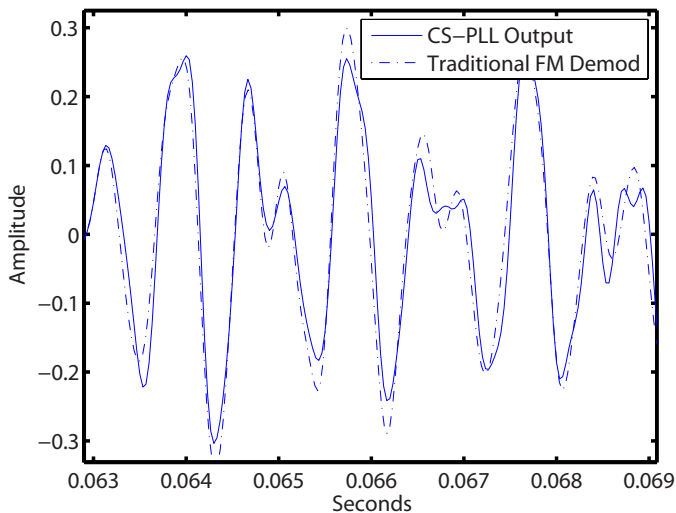


Fig. 6. CS-PLL output compared to a traditional FM demodulation of the Nyquist-rate samples.

B. Results

In Figure 5 we show the estimated power spectral density (PSD) at each stage in the WCR system. Note that in (b) the effect of the large interferer dominates the PSD. However when we cancel it first (see (c)), we are able to properly acquire the smaller SOIs. As shown, our original SNR of 40 dB is reduced to ≈ 25 dB, approximately a 15 dB loss. This is expected, as in theory undersampling by a factor of 20 should result in noise folding of approximately 13 dB (3 dB per octave (decimation by 2) for a total of $\log_2(20)$ octaves) [3]. This mirrors the effects of traditional downsampling and is not exclusive to compressive sensing. In (d) the signal at 175 MHz is kept and the other signals are filtered out.

The signal acquired after detection can then be supplied to the CS-PLL for demodulation. Note that the filtering and detection schemes are applied to time-limited blocks of data while the PLL works on a continuous sequence of samples. In our experiments above the block-length is not adequate to demonstrate the performance our CS-PLL, so we instead generate an input signal of 25 dB input SNR with the same parameters except only one FM signal present (and no TV interference) of significantly longer duration and then apply the CS-PLL. As Figure 6 indicates, the output of the CS-PLL closely resembles the result of traditional FM demodulation, even for the unduly pessimistic case of 25 dB input SNR (instead of 40 dB).

IV. CONCLUSION

The results in this paper demonstrate that an effective wideband radio receiver can be designed that simultaneously collects and processes signals from the entire bandwidth of interest. The undersampling nature of compressive sensing dictates that noise and interference energy folds into the various SOI bandwidths. However, by applying compressive-domain interference cancellation and filtering techniques before reconstruction we can prevent interference folding. Thus

we increase the output SNR of a particular SOI, thereby improving the utility of our system.

Signal classification was not directly addressed in this paper. Some work with matched filters, both for 1-D and 2-D signals, has appeared previously, but extending classification to the CS domain for FM receivers remains a point of future work [5].

The detection scheme used in this receiver model was very simple and likely not optimal. However, preliminary results indicate that a more sophisticated detection scheme capable of iteratively detecting and removing the largest signal can dramatically improve performance of the system.

Iterative methods are also potentially valuable in other parts of the system. For example, it may be useful to share information among the PLLs for several signals. For example the oscillator signal of one system may be useful in robustifying another feedback loop. The assumption of a relatively sparse spectrum particularly justifies sharing information among a few parallel devices.

V. ACKNOWLEDGEMENTS

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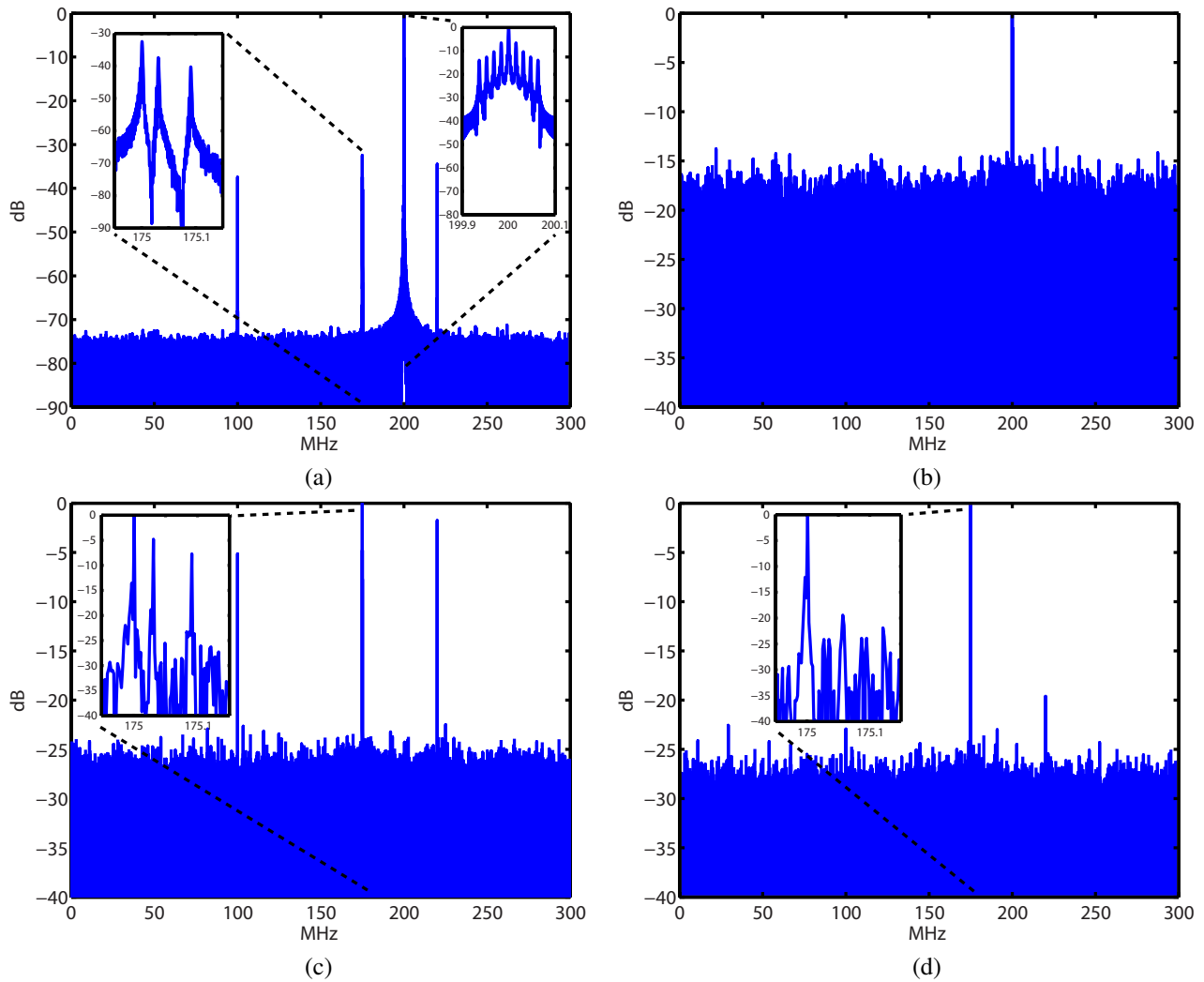


Fig. 5. Normalized power spectral densities (PSDs). (a) PSD of original Nyquist-rate signal. (b) PSD estimated from compressive measurements. (c) PSD estimated from compressive measurements after interference cancellation. (d) PSD estimated from compressive measurements after detection and filtering.

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